Reinforced Concrete Design | (8th Edition)

Chapter 1, Problem 8P (1 Bookmark) Show all steps: ON

Problem

Note: In the following problem, assume plain concrete to have a weight of 145 pcf (conservative) unless otherwise noted.

Rework Example 1–3 but invert the beam so that the flange is on the bottom and the web extends vertically upward. Calculate the cracking moment using the internal couple method and check using the flexure formula. Assume positive moment.

Example

Calculate the cracking moment (resisting moment) for the T-shaped unreinforced concrete beam shown in Figure 1–8. Use $f'_c = 4000$ psi. Assume positive moment (compression in the top). Use the internal couple method and check using the flexure formula.

Solution:

The neutral axis must be located so that the strain and stress diagrams may be defined. The location of the neutral axis with respect to the noted reference axis is calculated from

$$\bar{y} = \frac{\Sigma(Ay)}{\Sigma A} = \frac{4(20)(22) + 5(20)(10)}{4(20) + 5(20)} = 15.33 \text{ in.}$$

The bottom of the cross section is stressed in tension. Note that the stress at the bottom will be numerically larger than at the top because of the relative distances from the N.A. The stress at the bottom of the cross section will be set equal to the modulus of rupture (λ = 1.0 for normal-weight concrete):

$$f_{bott} = f_r = 7.5\lambda\sqrt{f'_c} = 7.5(1.0)\sqrt{4000} = 474 \text{ psi} = 0.474 \text{ ksi}$$

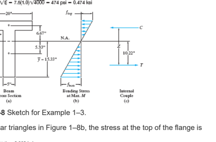


FIGURE 1-8 Sketch for Example 1–3.

Using similar triangles in Figure 1–8b, the stress at the top of the flange is

$$f_{top} = \frac{8.67}{15.33}(0.474) = 0.268 \text{ ksi}$$

Similarly, the stress at the bottom of the flange is

$$f_{bott\ of\ flange} = \frac{4.67}{15.33}(0.474) = 0.1444 \text{ ksi}$$

The total tensile force can be evaluated as follows:

$$T = \text{average stress} \times \text{area} = \tfrac{1}{2}(0.474)(15.33)(5) = 18.17 \text{ kips}$$

and its location below the N.A. is calculated from

$$\tfrac{2}{3}(15.33) = 10.22 \text{ in. (below the N.A.)}$$

The compressive force will be broken up into components because of the irregular area, as shown in Figure 1–9. Referring to both Figures 1–8 and 1–9, the component internal compressive forces, component internal couples, and *Mr* may now be evaluated. The component forces are first calculated:

$$\begin{aligned} C_1 &= 0.1444(20)(4) = 11.55 \text{ kips} \\ C_2 &= \tfrac{1}{2}(0.1236)(20)(4) = 4.94 \text{ kips} \\ C_3 &= \tfrac{1}{2}(0.1444)(5)(4.67) = 1.686 \text{ kips} \\ \text{total } C &= C_1 + C_2 + C_3 = 18.18 \text{ kips} \\ C &\approx T \quad \text{(O.K.)} \end{aligned}$$

Next we calculate the moment arm distance from each component compressive force to the tensile force *T*:

$$\begin{aligned} Z_1 &= 10.22 + 4.67 + \tfrac{1}{2}(4.00) = 16.89 \text{ in.} \\ Z_2 &= 10.22 + 4.67 + \tfrac{2}{3}(4.00) = 17.56 \text{ in.} \\ Z_3 &= 10.22 + \tfrac{2}{3}(4.67) = 13.33 \text{ in.} \end{aligned}$$

The magnitudes of the component internal couples are then calculated from force × moment arm as follows:

$$\begin{aligned} M_{R_1} &= 11.55(16.89) = 195.1 \text{ in.-kips} \\ M_{R_2} &= 4.94(17.56) = 86.7 \text{ in.-kips} \\ M_{R_3} &= 1.686(13.33) = 22.5 \text{ in.-kips} \\ M_{cr} &= M_R = M_{R_1} + M_{R_2} + M_{R_3} = 304 \text{ in.-kips} \end{aligned}$$

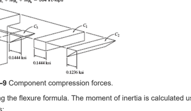


FIGURE 1–9 Component compression forces.

Check using the flexure formula. The moment of inertia is calculated using the transfer formula from statics:

$$\begin{aligned} I &= \Sigma I_o + \Sigma Ad^2 \\ I &= \tfrac{1}{12}(20)(4^3) + \tfrac{1}{12}(5)(20^3) + 4(20)(6.67^2) + 5(20)(5.33^2) \\ &= 9840 \text{ in.}^4 \end{aligned}$$

$$M_{cr} = M_R = \frac{f_r I}{c} = \frac{0.474(9840)}{15.33} = 304 \text{ in.-kips}$$

(Checks O.K.)

Step-by-step solution

Step 1 of 14

Calculate the position of the neutral axis with respect to the reference axis. Assume the reference axis at the bottom of the flange. Consider Figure (1).

$$\bar{y} = \frac{\sum(Ad)}{\sum A} = \frac{A_1X_1 + A_2X_2}{A_1 + A_2} = \frac{(b_1h_1)X_1 + (b_2h_2)X_2}{(b_1h_1) + (b_2h_2)}$$

Here, $\bar{y}$ is the distance of the neutral axis from the assumed reference axis, *A* is the area of the section, *d* is the distance from the centroid of the section to the assumed reference axis, A_1 and A_2 are the area of section 1 and section 2 respectively, d_1 and d_2 are the distance from the centroid of the section1 and section 2 to the assumed reference axis respectively, b_1 and b_2 are the width of section 1 and section 2 respectively, and h_1 and h_2 are the depth of section 1 and section 2 of the beam respectively.

Comment

Step 2 of 14

Substitute 5 in. for b_1, 20 in. for h_1, 20 in. for b_2, 4 in. for h_2, 14 in. for X_1, and 2 in. for X_2.

$$\bar{y} = \frac{[(5\text{in.}\times 20\text{in.})\times 14\text{in.}] + [(20\text{in.}\times 4\text{in.})\times 2\text{in.}]}{(5\text{in.}\times 20\text{in.}) + (20\text{in.}\times 4\text{in.})} = 8.67\text{in.}$$

Comment

Step 3 of 14

Thus, the neutral axis lies above 8.67 in. from the reference neutral axis.

Comment

Step 4 of 14

Calculate the modulus of rupture using ACI recommendations.

$$f_r = 7.5\lambda\sqrt{f'_c}$$

Here, f'_c is the strength of the concrete, λ is the modification factor, and f_r is the modulus of elasticity.

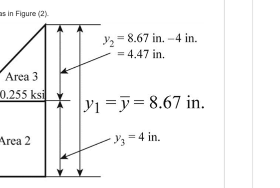
Substitute 4,000 psi for f'_c and 1 for λ.

$$f_r = 7.5\sqrt{4{,}000} = 474.34\text{ psi}\times\frac{1\text{kips}}{1{,}000\text{lb}} = 0.474\text{ksi}$$

Comment

Step 5 of 14

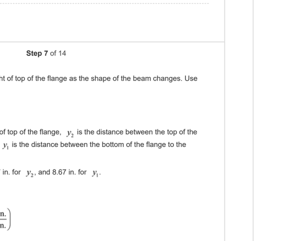
Divide the tension area of the stress block in to different areas due to irregular shape of the tension zone as in Figure (1).



Comment

Step 6 of 14

Show the proportional triangles as in Figure (2).



Comment

Step 7 of 14

Determine the stress at the height of top of the flange as the shape of the beam changes. Use the rule of proportional triangles.

$$\frac{x}{f_r} = \frac{y_2}{y_1}$$

Here, *x* is the stress at the level of top of the flange, y_2 is the distance between the top of the flange and the neutral axis, and y_1 is the distance between the bottom of the flange to the neutral axis.

Substitute 0.474 ksi for f_r, 4.67 in. for y_2, and 8.67 in. for y_1.

$$\frac{x}{0.474\text{ ksi}} = \frac{4.67\text{in.}}{8.67\text{in.}}$$
$$x = 0.474\text{ ksi}\times\left(\frac{4.67\text{in.}}{8.67\text{in.}}\right) = 0.255\text{ksi}$$

Comment

Step 8 of 14

Thus, determine the stress x_1.

$$x_1 = f_r - x$$

Substitute 0.255 ksi for *x* and 0.474 ksi for f_r.

$$x_1 = 0.474\text{ksi} - 0.255\text{ksi} = 0.219\text{ksi}$$

Tensile force T_1, T_2, and T_3 acts in the stress area 1, area 2, and area 3 respectively.

Determine the tensile force T_1.

$$T_1 = x_1 \times A_1 = x_1 \times\left(\frac{1}{2}h_2b_2\right)$$

Substitute 0.219 ksi for x_1, 4 in. for h_2, and 20 in for b_2.

$$T_1 = 0.219\text{ksi}\times\left(\frac{1}{2}\times 4\text{in.}\times 20\text{in.}\right) = 8.76\text{kips}$$

Comment

Step 9 of 14

Calculate the lever arm for the tensile force T_1.

$$L_1 = \left[\frac{2}{3}(D-\bar{y})\right] + \left(\bar{y} - \frac{1}{3}y_3\right)$$

Here, D is the total depth of the beam, y_3 is the height of the stress triangle 1, and *L* is the lever arm for the tensile force T_1.

Substitute 24 in. for *D*, 8.67 in. for $\bar{y}$, and 4 in. for y_3.

$$L_1 = \left[\frac{2}{3}(24\text{in.} - 8.67\text{in.})\right] + \left(8.67\text{in.} - \frac{1}{3}\times 4\text{in.}\right) = 17.56\text{in.}$$

Calculate the internal couple developed due to the tensile force T_1 and the compressive force *C*.

$$(\text{I.C})_1 = T_1 \times L_1$$

Here, $(\text{I.C})_1$ is the internal couple developed due to the tensile force T_1 and the compressive force *C*.

Substitute 8.76 kips for T_1 and 17.56 in. for L_1.

$$(\text{I.C})_1 = 8.76\text{kips}\times 17.56\text{in.} = 153.8\text{in.-kips}$$

Comment

Step 10 of 14

Similarly, calculate the tension forces, their lever arm, and the internal couple and tabulate the results as in Table (1).

Force	Magnitude (kips)	Lever arm (in.)	Internal couple (in.-kips)
T_1	8.76	17.56	153.8
T_2	20.4	16.89	344.6
T_3	2.89	12.33	39.7

Comment

Step 11 of 14

Calculate the creaking moment using the internal couple method.

$$M_{cr} = (\text{I.C})_1 + (\text{I.C})_2 + (\text{I.C})_3$$

Here, M_{cr} is the creaking moment.

Substitute 153.8 in.-kips for $(\text{I.C})_1$, 344.6 in.-kips for $(\text{I.C})_2$, and 39.7 in.-kips for $(\text{I.C})_3$.

$$M_{cr} = 153.8\text{ in.-kips} + 344.6\text{in.-kips} + 39.7\text{in.-kips} = 538\text{in.-kips}$$

Comment

Step 12 of 14

Calculate the moment of inertia of the beam.

$$I = \sum\left(\frac{bh^3}{12} + Ad^2\right) = \left(\frac{b_1h_1^3}{12} + (b_1h_1)d_1^2\right) + \left(\frac{b_2h_2^3}{12} + (b_2h_2)d_2^2\right)$$

Here, *I* is the combined moment of inertia, *b* is the width of the beam section, *h* is the height of the beam section, *A* is the area of the beam section, *d* is the distance between the ecntroid of the beam section to the neutral axis, b_1 and b_2 are the width of the beam section 1 and section 2 respectively, h_1 and h_2 are the height of the beam section 1 and beam section 2 respectively, and d_1 and d_2 are the distance from the centroid of section 1 and section 2 to the neutral axis respectively.

Comment

Step 13 of 14

Substitute 5 in. for b_1, 20 in. for h_1, 20 in. for b_2, 4 in. for h_2, 5.33 in. for d_1, and 6.67 in. for d_2.

$$I = \left\{\begin{array}{l}\left[\dfrac{5\text{in.}\times(20\text{in.})^3}{12} + (5\text{in.}\times 20\text{in.})\times(5.33\text{in.})^2\right] \\ + \left[\dfrac{20\text{in.}\times(4\text{in.})^3}{12} + (4\text{in.}\times 20\text{in.})\times(6.67\text{in.})^2\right]\end{array}\right\} = 9{,}840\text{in.}^4$$

The distance from the neutral axis of the section to the bottom tension fiber (*c*) is same as the distance between the neutral axis and the assumed reference axis.

Therefore,

$$c = \bar{y}$$

Substitute 8.67 in. for $\bar{y}$.

$$c = 8.67\text{in.}$$

Comment

Step 14 of 14

Calculate the critical moment using the flexure formula.

$$M_{cr} = \frac{f_r I}{c}$$

Here, *c* is the distance from the neutral axis to the outside tension or compression fiber of the beam.

Substitute 0.474 ksi for f_r, 8.67 in. for *c*, and $9{,}840\text{in.}^4$ for *I*.

$$M_{cr} = \frac{0.474\text{ksi}\times 9{,}840\text{in.}^4}{8.67\text{in.}} = 538\text{in.-kips}$$

The maximum bending stress calculated using the internal couple method is equal to the maximum bending stress calculated using the flexure formula. Thus, the check is O.K.

Thus,

The critical bending moment calculated using internal couple method is $\boxed{538\text{in.-kips}}$.

The critical bending moment calculated using the flexure formula is $\boxed{538\text{in.-kips}}$.

Comment

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Chapter 1, Problem 4P
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Chapter 1, Problem 5P
Note: In the following problem, assume plain concrete to have a weight of 145 pcf (conservative) unless otherwise noted.The normal-weight plain concrete beam shown is on a simple span of 10 ft. It carries a dead load (which includes the weight...
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