Reinforced Concrete Design | 6th Edition

Chapter 11 Solutions Key Exercises Flexure Problems Chapter 11 Solutions Key

Flexure Problems

Problems in this chapter test your ability to analyze and design reinforced concrete beams for flexure using strength design principles. Work through each problem step by step, showing all formulas, substitutions, and units. Check your answers against the solutions provided, but try to solve each problem on your own first before looking ahead.

Pay special attention to problems that require you to determine whether a beam is tension-controlled, compression-controlled, or in the transition zone. This classification affects the strength reduction factor ϕ and can significantly change the design capacity. Also watch for problems that require you to check minimum reinforcement limits, as these often govern the design of lightly loaded members.

Problem 11.1

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 12$ in, $d = 20$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 8 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.1

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 0.79 = 3.16 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.16 \times 60{,}000}{0.85 \times 4{,}000 \times 12} = \frac{189{,}600}{40{,}800} = 4.65 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3.16 \times 60 \times \left(20 - \frac{4.65}{2}\right) = 189.6 \times 17.68 = 3{,}352 \text{ in-kips}$$

$$M_n = 279.3 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.65 / 0.85 = 5.47$ in.

$$\varepsilon_t = \frac{0.003(20 - 5.47)}{5.47} = \frac{0.003 \times 14.53}{5.47} = 0.00797$$

Since $\varepsilon_t = 0.00797 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 279.3 = 251.4 \text{ ft-kips}$$

Problem 11.2

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 14$ in, $d = 22$ in, $f'_c = 5{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with six No. 9 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.2

Step 1: Calculate the area of steel reinforcement.

$$A_s = 6 \times 1.00 = 6.00 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{6.00 \times 60{,}000}{0.85 \times 5{,}000 \times 14} = \frac{360{,}000}{59{,}500} = 6.05 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 6.00 \times 60 \times \left(22 - \frac{6.05}{2}\right) = 360 \times 18.97 = 6{,}829 \text{ in-kips}$$

$$M_n = 569.1 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 6.05 / 0.85 = 7.12$ in.

$$\varepsilon_t = \frac{0.003(22 - 7.12)}{7.12} = \frac{0.003 \times 14.88}{7.12} = 0.00627$$

Since $\varepsilon_t = 0.00627 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 569.1 = 512.2 \text{ ft-kips}$$

Problem 11.3

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 10$ in, $d = 18$ in, $f'_c = 3{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with three No. 7 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.3

Step 1: Calculate the area of steel reinforcement.

$$A_s = 3 \times 0.60 = 1.80 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1.80 \times 60{,}000}{0.85 \times 3{,}000 \times 10} = \frac{108{,}000}{25{,}500} = 4.24 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1.80 \times 60 \times \left(18 - \frac{4.24}{2}\right) = 108 \times 15.88 = 1{,}715 \text{ in-kips}$$

$$M_n = 142.9 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.24 / 0.85 = 4.99$ in.

$$\varepsilon_t = \frac{0.003(18 - 4.99)}{4.99} = \frac{0.003 \times 13.01}{4.99} = 0.00782$$

Since $\varepsilon_t = 0.00782 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 142.9 = 128.6 \text{ ft-kips}$$

Problem 11.4

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 16$ in, $d = 24$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with eight No. 8 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.4

Step 1: Calculate the area of steel reinforcement.

$$A_s = 8 \times 0.79 = 6.32 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{6.32 \times 60{,}000}{0.85 \times 4{,}000 \times 16} = \frac{379{,}200}{54{,}400} = 6.97 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 6.32 \times 60 \times \left(24 - \frac{6.97}{2}\right) = 379.2 \times 20.52 = 7{,}781 \text{ in-kips}$$

$$M_n = 648.4 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 6.97 / 0.85 = 8.20$ in.

$$\varepsilon_t = \frac{0.003(24 - 8.20)}{8.20} = \frac{0.003 \times 15.80}{8.20} = 0.00578$$

Since $\varepsilon_t = 0.00578 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 648.4 = 583.6 \text{ ft-kips}$$

Problem 11.5

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 12$ in, $d = 16$ in, $f'_c = 3{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with two No. 9 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.5

Step 1: Calculate the area of steel reinforcement.

$$A_s = 2 \times 1.00 = 2.00 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.00 \times 60{,}000}{0.85 \times 3{,}000 \times 12} = \frac{120{,}000}{30{,}600} = 3.92 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2.00 \times 60 \times \left(16 - \frac{3.92}{2}\right) = 120 \times 14.04 = 1{,}685 \text{ in-kips}$$

$$M_n = 140.4 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 3.92 / 0.85 = 4.61$ in.

$$\varepsilon_t = \frac{0.003(16 - 4.61)}{4.61} = \frac{0.003 \times 11.39}{4.61} = 0.00741$$

Since $\varepsilon_t = 0.00741 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 140.4 = 126.4 \text{ ft-kips}$$

Problem 11.6

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 18$ in, $d = 28$ in, $f'_c = 5{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 10 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.6

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 1.27 = 5.08 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5.08 \times 60{,}000}{0.85 \times 5{,}000 \times 18} = \frac{304{,}800}{76{,}500} = 3.98 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 5.08 \times 60 \times \left(28 - \frac{3.98}{2}\right) = 304.8 \times 26.01 = 7{,}926 \text{ in-kips}$$

$$M_n = 660.5 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 3.98 / 0.85 = 4.68$ in.

$$\varepsilon_t = \frac{0.003(28 - 4.68)}{4.68} = \frac{0.003 \times 23.32}{4.68} = 0.01495$$

Since $\varepsilon_t = 0.01495 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 660.5 = 594.5 \text{ ft-kips}$$

Problem 11.7

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 14$ in, $d = 22$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with six No. 8 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.7

Step 1: Calculate the area of steel reinforcement.

$$A_s = 6 \times 0.79 = 4.74 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.74 \times 60{,}000}{0.85 \times 4{,}000 \times 14} = \frac{284{,}400}{47{,}600} = 5.98 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4.74 \times 60 \times \left(22 - \frac{5.98}{2}\right) = 284.4 \times 19.01 = 5{,}407 \text{ in-kips}$$

$$M_n = 450.6 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 5.98 / 0.85 = 7.04$ in.

$$\varepsilon_t = \frac{0.003(22 - 7.04)}{7.04} = \frac{0.003 \times 14.96}{7.04} = 0.00638$$

Since $\varepsilon_t = 0.00638 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 450.6 = 405.5 \text{ ft-kips}$$

Problem 11.8

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 12$ in, $d = 18$ in, $f'_c = 3{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 7 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.8

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 0.60 = 2.40 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.40 \times 60{,}000}{0.85 \times 3{,}000 \times 12} = \frac{144{,}000}{30{,}600} = 4.71 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2.40 \times 60 \times \left(18 - \frac{4.71}{2}\right) = 144 \times 15.65 = 2{,}254 \text{ in-kips}$$

$$M_n = 187.8 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.71 / 0.85 = 5.54$ in.

$$\varepsilon_t = \frac{0.003(18 - 5.54)}{5.54} = \frac{0.003 \times 12.46}{5.54} = 0.00675$$

Since $\varepsilon_t = 0.00675 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 187.8 = 169.0 \text{ ft-kips}$$

Problem 11.9

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 20$ in, $d = 30$ in, $f'_c = 5{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with eight No. 9 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.9

Step 1: Calculate the area of steel reinforcement.

$$A_s = 8 \times 1.00 = 8.00 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{8.00 \times 60{,}000}{0.85 \times 5{,}000 \times 20} = \frac{480{,}000}{85{,}000} = 5.65 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 8.00 \times 60 \times \left(30 - \frac{5.65}{2}\right) = 480 \times 27.18 = 13{,}046 \text{ in-kips}$$

$$M_n = 1{,}087 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 5.65 / 0.85 = 6.65$ in.

$$\varepsilon_t = \frac{0.003(30 - 6.65)}{6.65} = \frac{0.003 \times 23.35}{6.65} = 0.01053$$

Since $\varepsilon_t = 0.01053 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 1{,}087 = 978.3 \text{ ft-kips}$$

Problem 11.10

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 16$ in, $d = 26$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with six No. 10 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.10

Step 1: Calculate the area of steel reinforcement.

$$A_s = 6 \times 1.27 = 7.62 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{7.62 \times 60{,}000}{0.85 \times 4{,}000 \times 16} = \frac{457{,}200}{54{,}400} = 8.41 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 7.62 \times 60 \times \left(26 - \frac{8.41}{2}\right) = 457.2 \times 21.80 = 9{,}968 \text{ in-kips}$$

$$M_n = 830.7 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 8.41 / 0.85 = 9.89$ in.

$$\varepsilon_t = \frac{0.003(26 - 9.89)}{9.89} = \frac{0.003 \times 16.11}{9.89} = 0.00489$$

Since $\varepsilon_t = 0.00489 < 0.005$, the section is in the transition zone between tension- and compression-controlled behavior. The strength reduction factor must be determined by linear interpolation between $\phi = 0.65$ for compression control and $\phi = 0.90$ for tension control.

$$\phi = 0.65 + 0.25 \left(\frac{\varepsilon_t - 0.002}{0.003}\right) = 0.65 + 0.25 \left(\frac{0.00489 - 0.002}{0.003}\right) = 0.65 + 0.25 \times 0.830 = 0.858$$

$$\phi M_n = 0.858 \times 830.7 = 712.7 \text{ ft-kips}$$

Problem 11.11

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 12$ in, $d = 20$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 8 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.11

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 0.79 = 3.16 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.16 \times 60{,}000}{0.85 \times 4{,}000 \times 12} = \frac{189{,}600}{40{,}800} = 4.65 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3.16 \times 60 \times \left(20 - \frac{4.65}{2}\right) = 189.6 \times 17.68 = 3{,}352 \text{ in-kips}$$

$$M_n = 279.3 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.65 / 0.85 = 5.47$ in.

$$\varepsilon_t = \frac{0.003(20 - 5.47)}{5.47} = \frac{0.003 \times 14.53}{5.47} = 0.00797$$

Since $\varepsilon_t = 0.00797 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 279.3 = 251.4 \text{ ft-kips}$$

Problem 11.12

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 10$ in, $d = 16$ in, $f'_c = 3{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with three No. 7 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.12

Step 1: Calculate the area of steel reinforcement.

$$A_s = 3 \times 0.60 = 1.80 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1.80 \times 60{,}000}{0.85 \times 3{,}000 \times 10} = \frac{108{,}000}{25{,}500} = 4.24 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 1.80 \times 60 \times \left(16 - \frac{4.24}{2}\right) = 108 \times 13.88 = 1{,}499 \text{ in-kips}$$

$$M_n = 124.9 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.24 / 0.85 = 4.99$ in.

$$\varepsilon_t = \frac{0.003(16 - 4.99)}{4.99} = \frac{0.003 \times 11.01}{4.99} = 0.00662$$

Since $\varepsilon_t = 0.00662 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 124.9 = 112.4 \text{ ft-kips}$$

Problem 11.13

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 14$ in, $d = 24$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with six No. 8 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.13

Step 1: Calculate the area of steel reinforcement.

$$A_s = 6 \times 0.79 = 4.74 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.74 \times 60{,}000}{0.85 \times 4{,}000 \times 14} = \frac{284{,}400}{47{,}600} = 5.98 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4.74 \times 60 \times \left(24 - \frac{5.98}{2}\right) = 284.4 \times 21.01 = 5{,}976 \text{ in-kips}$$

$$M_n = 498.0 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 5.98 / 0.85 = 7.04$ in.

$$\varepsilon_t = \frac{0.003(24 - 7.04)}{7.04} = \frac{0.003 \times 16.96}{7.04} = 0.00723$$

Since $\varepsilon_t = 0.00723 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 498.0 = 448.2 \text{ ft-kips}$$

Problem 11.14

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 18$ in, $d = 28$ in, $f'_c = 5{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 10 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.14

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 1.27 = 5.08 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5.08 \times 60{,}000}{0.85 \times 5{,}000 \times 18} = \frac{304{,}800}{76{,}500} = 3.98 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 5.08 \times 60 \times \left(28 - \frac{3.98}{2}\right) = 304.8 \times 26.01 = 7{,}926 \text{ in-kips}$$

$$M_n = 660.5 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 3.98 / 0.85 = 4.68$ in.

$$\varepsilon_t = \frac{0.003(28 - 4.68)}{4.68} = \frac{0.003 \times 23.32}{4.68} = 0.01495$$

Since $\varepsilon_t = 0.01495 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 660.5 = 594.5 \text{ ft-kips}$$

Problem 11.15

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 12$ in, $d = 20$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 8 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.15

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 0.79 = 3.16 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.16 \times 60{,}000}{0.85 \times 4{,}000 \times 12} = \frac{189{,}600}{40{,}800} = 4.65 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3.16 \times 60 \times \left(20 - \frac{4.65}{2}\right) = 189.6 \times 17.68 = 3{,}352 \text{ in-kips}$$

$$M_n = 279.3 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.65 / 0.85 = 5.47$ in.

$$\varepsilon_t = \frac{0.003(20 - 5.47)}{5.47} = \frac{0.003 \times 14.53}{5.47} = 0.00797$$

Since $\varepsilon_t = 0.00797 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 279.3 = 251.4 \text{ ft-kips}$$

Problem 11.16

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 14$ in, $d = 22$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with six No. 8 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.16

Step 1: Calculate the area of steel reinforcement.

$$A_s = 6 \times 0.79 = 4.74 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.74 \times 60{,}000}{0.85 \times 4{,}000 \times 14} = \frac{284{,}400}{47{,}600} = 5.98 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4.74 \times 60 \times \left(22 - \frac{5.98}{2}\right) = 284.4 \times 19.01 = 5{,}407 \text{ in-kips}$$

$$M_n = 450.6 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 5.98 / 0.85 = 7.04$ in.

$$\varepsilon_t = \frac{0.003(22 - 7.04)}{7.04} = \frac{0.003 \times 14.96}{7.04} = 0.00638$$

Since $\varepsilon_t = 0.00638 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 450.6 = 405.5 \text{ ft-kips}$$

Problem 11.17

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 12$ in, $d = 18$ in, $f'_c = 3{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 7 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.17

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 0.60 = 2.40 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.40 \times 60{,}000}{0.85 \times 3{,}000 \times 12} = \frac{144{,}000}{30{,}600} = 4.71 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2.40 \times 60 \times \left(18 - \frac{4.71}{2}\right) = 144 \times 15.65 = 2{,}254 \text{ in-kips}$$

$$M_n = 187.8 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.71 / 0.85 = 5.54$ in.

$$\varepsilon_t = \frac{0.003(18 - 5.54)}{5.54} = \frac{0.003 \times 12.46}{5.54} = 0.00675$$

Since $\varepsilon_t = 0.00675 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 187.8 = 169.0 \text{ ft-kips}$$

Problem 11.18

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 20$ in, $d = 30$ in, $f'_c = 5{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with eight No. 9 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.18

Step 1: Calculate the area of steel reinforcement.

$$A_s = 8 \times 1.00 = 8.00 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{8.00 \times 60{,}000}{0.85 \times 5{,}000 \times 20} = \frac{480{,}000}{85{,}000} = 5.65 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 8.00 \times 60 \times \left(30 - \frac{5.65}{2}\right) = 480 \times 27.18 = 13{,}046 \text{ in-kips}$$

$$M_n = 1{,}087 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 5.65 / 0.85 = 6.65$ in.

$$\varepsilon_t = \frac{0.003(30 - 6.65)}{6.65} = \frac{0.003 \times 23.35}{6.65} = 0.01053$$

Since $\varepsilon_t = 0.01053 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 1{,}087 = 978.3 \text{ ft-kips}$$

Problem 11.19

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 16$ in, $d = 26$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with six No. 10 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.19

Step 1: Calculate the area of steel reinforcement.

$$A_s = 6 \times 1.27 = 7.62 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{7.62 \times 60{,}000}{0.85 \times 4{,}000 \times 16} = \frac{457{,}200}{54{,}400} = 8.41 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 7.62 \times 60 \times \left(26 - \frac{8.41}{2}\right) = 457.2 \times 21.80 = 9{,}968 \text{ in-kips}$$

$$M_n = 830.7 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 8.41 / 0.85 = 9.89$ in.

$$\varepsilon_t = \frac{0.003(26 - 9.89)}{9.89} = \frac{0.003 \times 16.11}{9.89} = 0.00489$$

Since $\varepsilon_t = 0.00489 < 0.005$, the section is in the transition zone between tension- and compression-controlled behavior. The strength reduction factor must be determined by linear interpolation between $\phi = 0.65$ for compression control and $\phi = 0.90$ for tension control.

$$\phi = 0.65 + 0.25 \left(\frac{\varepsilon_t - 0.002}{0.003}\right) = 0.65 + 0.25 \left(\frac{0.00489 - 0.002}{0.003}\right) = 0.65 + 0.25 \times 0.830 = 0.858$$

$$\phi M_n = 0.858 \times 830.7 = 712.7 \text{ ft-kips}$$

Problem 11.20

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 14$ in, $d = 22$ in, $f'_c = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with six No. 8 bars in two layers. Assume tension-controlled behavior and verify your assumption.

Solution 11.20

Step 1: Calculate the area of steel reinforcement.

$$A_s = 6 \times 0.79 = 4.74 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.74 \times 60{,}000}{0.85 \times 4{,}000 \times 14} = \frac{284{,}400}{47{,}600} = 5.98 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 4.74 \times 60 \times \left(22 - \frac{5.98}{2}\right) = 284.4 \times 19.01 = 5{,}407 \text{ in-kips}$$

$$M_n = 450.6 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 5.98 / 0.85 = 7.04$ in.

$$\varepsilon_t = \frac{0.003(22 - 7.04)}{7.04} = \frac{0.003 \times 14.96}{7.04} = 0.00638$$

Since $\varepsilon_t = 0.00638 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 450.6 = 405.5 \text{ ft-kips}$$

Problem 11.21

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 12$ in, $d = 18$ in, $f'_c = 3{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 7 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.21

Step 1: Calculate the area of steel reinforcement.

$$A_s = 4 \times 0.60 = 2.40 \text{ in}^2$$

Step 2: Calculate the depth of the equivalent rectangular stress block.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.40 \times 60{,}000}{0.85 \times 3{,}000 \times 12} = \frac{144{,}000}{30{,}600} = 4.71 \text{ in}$$

Step 3: Calculate the nominal moment strength.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 2.40 \times 60 \times \left(18 - \frac{4.71}{2}\right) = 144 \times 15.65 = 2{,}254 \text{ in-kips}$$

$$M_n = 187.8 \text{ ft-kips}$$

Step 4: Check the assumption of tension control. The net tensile strain in the extreme tension steel is:

$$\varepsilon_t = \frac{0.003(d - c)}{c}$$

where $c = a / \beta_1 = 4.71 / 0.85 = 5.54$ in.

$$\varepsilon_t = \frac{0.003(18 - 5.54)}{5.54} = \frac{0.003 \times 12.46}{5.54} = 0.00675$$

Since $\varepsilon_t = 0.00675 > 0.005$, the section is tension-controlled and $\phi = 0.90$.

$$\phi M_n = 0.90 \times 187.8 = 169.0 \text{ ft-kips}$$

Problem 11.22

Determine the nominal moment strength M_n and the design moment strength ϕM_n for a rectangular beam with $b = 18$ in, $d = 28$ in, $f'_c = 5{,}000$ psi, and $f_y = 60{,}000$ psi. The beam is reinforced with four No. 10 bars in a single layer. Assume tension-controlled behavior and verify your assumption.

Solution 11.22

Step 1: Calcul