Mechanical Behavior of Materials | (2nd Edition)

Step-by-step solution

Step 1 of 10

Calculate the interplanar spacing for the fracture plane $\{0001\}$.

$$d=\frac{1}{\sqrt{\frac{4}{3a^2}\left(h^2+k^2+hk\right)+\frac{l^2}{c^2}}}$$

Here, the index notation of *x* direction is *h*, index notation of *y* direction is *k*, index notation of *k* direction is *i*, index notation of *z* direction is *l*, and lattice parameter is *a*.

Consider the lattice parameter constants *a* and *c* of sapphire at room temperature as $4.7592\ \mathring{\text{A}}$ and $12.9915\ \mathring{\text{A}}$.

Substitute 0 for *h*, 0 for *k*, 1 for *l*, $4.7592\ \mathring{\text{A}}$ for *a*, $12.9915\ \mathring{\text{A}}$ for *c*.

$$\begin{aligned}d&=\frac{1}{\sqrt{\frac{4}{3\left(4.7592\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}\left((0)^2+(0)^2+(0)\right)+\frac{(1)^2}{\left(12.9915\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}}}\\&=\frac{1}{5.924\times10^{17}}\\&=1.6877\times10^{-18}\ \text{m}\end{aligned}$$

Comment

Step 2 of 10

Calculate the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{0001\}$.

$$\sigma_c=\sqrt{\frac{E\gamma}{d}}$$

Here, young's modulus is *E*, fracture surface energy is γ, and interplanar spacing is *d*.

From the modulus of elasticity of ceramic materials table, select the young's modulus of alumina as $365\ \text{GPa}$.

From the fracture surface energy of sapphire at room temperature table, select the fracture surface energy of fracture plane $\{0001\}$ as $42\ \text{J/m}^2$.

Substitute $365\ \text{GPa}$ for *E*, $42\ \text{J/m}^2$ for γ, and $1.6877\times10^{-18}\ \text{m}$ for *d*.

$$\begin{aligned}\sigma_c&=\sqrt{\frac{(365\ \text{GPa})(42\ \text{J/m}^2)}{1.6877\times10^{-18}\ \text{m}}}\\&=\sqrt{\frac{(365\ \text{GPa})(42\ \text{J/m}^2)}{1.6877\times10^{-18}\ \text{m}}\left(\frac{1\ \text{N/m}^2}{1\ \text{Pa}}\right)\left(\frac{1\ \text{N m}}{1\ \text{J}}\right)}\\&=30.13\times10^5\ \text{GN/m}^2\left(\frac{1\ \text{GPa}}{1\ \text{GN/m}^2}\right)\\&=30.13\times10^5\ \text{GPa}\end{aligned}$$

Hence, the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{0001\}$ is $\boxed{30.13\times10^5\ \text{GPa}}$.

Comment

Step 3 of 10

Calculate the interplanar spacing for the fracture plane $\{1010\}$.

$$d=\frac{1}{\sqrt{\frac{4}{3a^2}\left(h^2+k^2+hk\right)+\frac{l^2}{c^2}}}$$

Here, the index notation of *x* direction is *h*, index notation of *y* direction is *k*, index notation of *k* direction is *i*, index notation of *z* direction is *l*, and lattice parameter is *a*.

Substitute 1 for *h*, 0 for *k*, 0 for *l*, $4.7592\ \mathring{\text{A}}$ for *a*, $12.9915\ \mathring{\text{A}}$ for *c*.

$$\begin{aligned}d&=\frac{1}{\sqrt{\frac{4}{3\left(4.7592\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}\left((1)^2+(0)^2+(0)\right)+\frac{(0)^2}{\left(12.9915\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}}}\\&=\frac{1}{2.426\times10^{9}}\\&=4.121\times10^{-10}\ \text{m}\end{aligned}$$

Comment

Step 4 of 10

Calculate the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{1010\}$.

$$\sigma_c=\sqrt{\frac{E\gamma}{d}}$$

Here, young's modulus is *E*, fracture surface energy is γ, and interplanar spacing is *d*.

From the fracture surface energy of sapphire at room temperature table, select the fracture surface energy of fracture plane $\{1010\}$ as $7.3\ \text{J/m}^2$.

Comment

Step 5 of 10

Substitute $365\ \text{GPa}$ for *E*, $7.3\ \text{J/m}^2$ for γ, and $4.121\times10^{-10}\ \text{m}$ for *d*.

$$\begin{aligned}\sigma_c&=\sqrt{\frac{(365\ \text{GPa})(7.3\ \text{J/m}^2)}{4.121\times10^{-10}\ \text{m}}}\\&=\sqrt{\frac{(365\ \text{GPa})(7.3\ \text{J/m}^2)}{4.121\times10^{-10}\ \text{m}}\left(\frac{10^9\ \text{N/m}^2}{1\ \text{GPa}}\right)\left(\frac{1\ \text{N m}}{1\ \text{J}}\right)}\\&=80.4\times10^9\ \text{N/m}^2\left(\frac{1\ \text{GPa}}{10^9\ \text{N/m}^2}\right)\\&=80.4\ \text{GPa}\end{aligned}$$

Hence, the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{1010\}$ is $\boxed{80.4\ \text{GPa}}$.

Comment

Step 6 of 10

Calculate the interplanar spacing for the fracture plane $\{1012\}$.

$$d=\frac{1}{\sqrt{\frac{4}{3a^2}\left(h^2+k^2+hk\right)+\frac{l^2}{c^2}}}$$

Here, the index notation of *x* direction is *h*, index notation of *y* direction is *k*, index notation of *k* direction is *i*, index notation of *z* direction is *l*, and lattice parameter is *a*.

Substitute 1 for *h*, 0 for *k*, 2 for *l*, $4.7592\ \mathring{\text{A}}$ for *a*, $12.9915\ \mathring{\text{A}}$ for *c*.

$$\begin{aligned}d&=\frac{1}{\sqrt{\frac{4}{3\left(4.7592\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}\left((1)^2+(0)^2+(0)\right)+\frac{(2)^2}{\left(12.9915\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}}}\\&=\frac{1}{2.873\times10^{9}}\\&=3.48\times10^{-10}\ \text{m}\end{aligned}$$

Comment

Step 7 of 10

Calculate the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{1012\}$.

$$\sigma_c=\sqrt{\frac{E\gamma}{d}}$$

Here, young's modulus is *E*, fracture surface energy is γ, and interplanar spacing is *d*.

From the fracture surface energy of sapphire at room temperature table, select the fracture surface energy of fracture plane $\{1012\}$ as $6\ \text{J/m}^2$.

Substitute $365\ \text{GPa}$ for *E*, $6\ \text{J/m}^2$ for γ, and $3.48\times10^{-10}\ \text{m}$ for *d*.

$$\begin{aligned}\sigma_c&=\sqrt{\frac{(365\ \text{GPa})(6\ \text{J/m}^2)}{3.48\times10^{-10}\ \text{m}}}\\&=\sqrt{\frac{(365\ \text{GPa})(6\ \text{J/m}^2)}{3.48\times10^{-10}\ \text{m}}\left(\frac{10^9\ \text{N/m}^2}{1\ \text{GPa}}\right)\left(\frac{1\ \text{N m}}{1\ \text{J}}\right)}\\&=79.3\times10^9\ \text{N/m}^2\left(\frac{1\ \text{GPa}}{10^9\ \text{N/m}^2}\right)\\&=79.3\ \text{GPa}\end{aligned}$$

Hence, the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{1012\}$ is $\boxed{79.3\ \text{GPa}}$.

Comment

Step 8 of 10

Calculate the interplanar spacing for the fracture plane $\{1126\}$.

$$d=\frac{1}{\sqrt{\frac{4}{3a^2}\left(h^2+k^2+hk\right)+\frac{l^2}{c^2}}}$$

Here, the index notation of *x* direction is *h*, index notation of *y* direction is *k*, index notation of *k* direction is *i*, index notation of *z* direction is *l*, and lattice parameter is *a*.

Substitute 1 for *h*, 1 for *k*, 6 for *l*, $4.7592\ \mathring{\text{A}}$ for *a*, $12.9915\ \mathring{\text{A}}$ for *c*.

$$\begin{aligned}d&=\frac{1}{\sqrt{\frac{4}{3\left(4.7592\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}\left((1)^2+(1)^2+(1)(1)\right)+\frac{(6)^2}{\left(12.9915\ \mathring{\text{A}}\left(\frac{10^{-10}\ \text{m}}{1\ \mathring{\text{A}}}\right)\right)^2}}}\\&=\frac{1}{6.244\times10^{9}}\\&=1.601\times10^{-10}\ \text{m}\end{aligned}$$

Comment

Step 9 of 10

Calculate the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{1126\}$.

$$\sigma_c=\sqrt{\frac{E\gamma}{d}}$$

Here, young's modulus is *E*, fracture surface energy is γ, and interplanar spacing is *d*.

Comment

Step 10 of 10

From the fracture surface energy of sapphire at room temperature table, select the fracture surface energy of fracture plane $\{1126\}$ as $24.4\ \text{J/m}^2$.

Substitute $365\ \text{GPa}$ for *E*, $24.4\ \text{J/m}^2$ for γ, and $1.601\times10^{-10}\ \text{m}$ for *d*.

$$\begin{aligned}\sigma_c&=\sqrt{\frac{(365\ \text{GPa})(24.4\ \text{J/m}^2)}{1.601\times10^{-10}\ \text{m}}}\\&=\sqrt{\frac{(365\ \text{GPa})(24.4\ \text{J/m}^2)}{1.601\times10^{-10}\ \text{m}}\left(\frac{10^9\ \text{N/m}^2}{1\ \text{GPa}}\right)\left(\frac{1\ \text{N m}}{1\ \text{J}}\right)}\\&=235.8\times10^9\ \text{N/m}^2\left(\frac{1\ \text{GPa}}{10^9\ \text{N/m}^2}\right)\\&=235.8\ \text{GPa}\end{aligned}$$

Hence, the theoretical cleavage stress (σ_c) for sapphire along its crystallographic orientations of fracture plane $\{1126\}$ is $\boxed{235.8\ \text{GPa}}$.

Comment

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Calculate the length of winged crack . Here, the length of crack is , and crack size radius is a. Consider the length...

See solution

(L)

Chapter 8, Solution 4E

See solution

Express the fracture tou

$$\begin{aligned}K_{Ic}&=\sigma\sqrt{\pi a}\\&=\sigma\sqrt{\frac{\pi D}{2}}\end{aligned}$$

Here, the fracture stress

Calculate the fracture st

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