

12-1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6) \text{ m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6 \text{ s}$, and what is its position when $t = 11 \text{ s}$?

SOLUTION

$$a = 2t - 6$$

$$dv = a \, dt$$

$$\int_0^v dv = \int_0^t (2t - 6) \, dt$$

$$v = t^2 - 6t$$

$$ds = v \, dt$$

$$\int_0^s ds = \int_0^t (t^2 - 6t) \, dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6 \text{ s}$,

$$v = 0$$

Ans.

When $t = 11 \text{ s}$,

$$s = 80.7 \text{ m}$$

Ans.

Ans:
 $v = 0$
 $s = 80.7 \text{ m}$

12-2.

The acceleration of a particle as it moves along a straight line is given by $a = (4t^3 - 1) \text{ m/s}^2$, where t is in seconds. If $s = 2 \text{ m}$ and $v = 5 \text{ m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 5 \text{ s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$\int_5^v dv = \int_0^t (4t^3 - 1) dt$$

$$v = t^4 - t + 5$$

$$\int_2^s ds = \int_0^t (t^4 - t + 5) dt$$

$$s = \frac{1}{5}t^5 - \frac{1}{2}t^2 + 5t + 2$$

When $t = 5 \text{ s}$,

$$v = 625 \text{ m/s}$$

Ans.

$$s = 639.5 \text{ m}$$

Ans.

Since $v \neq 0$ then

$$d = 639.5 - 2 = 637.5 \text{ m}$$

Ans.

Ans:

$$v = 625 \text{ m/s}$$

$$s = 639.5 \text{ m}$$

$$d = 637.5 \text{ m}$$

12-3.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

SOLUTION

$$v = 6t - 3t^2$$

$$a = \frac{dv}{dt} = 6 - 6t$$

$$\text{At } t = 3 \text{ s}$$

$$a = -12 \text{ m/s}^2$$

$$ds = v dt$$

$$\int_0^s ds = \int_0^t (6t - 3t^2) dt$$

$$s = 3t^2 - t^3$$

$$\text{At } t = 3 \text{ s}$$

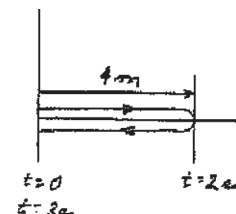
$$s = 0$$

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

$$\text{when } t = 2 \text{ s, } s = 3(2)^2 - (2)^3 = 4 \text{ m}$$

$$s_T = 4 + 4 = 8 \text{ m}$$

$$(v_{sp})_{\text{avg}} = \frac{s_T}{t} = \frac{8}{3} = 2.67 \text{ m/s}$$

Ans.**Ans.****Ans.****Ans.****Ans:**

$$a = -12 \text{ m/s}^2$$

$$s = 0$$

$$s_T = 8 \text{ m}$$

$$(v_{sp})_{\text{avg}} = 2.67 \text{ m/s}$$

***12–4.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$$s = 10t^2 + 20$$

$$(a) \quad s|_{1\text{ s}} = 10(1)^2 + 20 = 30 \text{ mm}$$

$$s|_{5\text{ s}} = 10(5)^2 + 20 = 270 \text{ mm}$$

$$\Delta s = 270 - 30 = 240 \text{ mm}$$

Ans.

$$(b) \quad \Delta t = 5 - 1 = 4 \text{ s}$$

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{240}{4} = 60 \text{ mm/s}$$

Ans.

$$(c) \quad a = \frac{d^2s}{dt^2} = 20 \text{ mm/s}^2 \quad (\text{for all } t)$$

Ans.

Ans:

$$\Delta s = 240 \text{ mm}$$

$$v_{avg} = 60 \text{ mm/s}$$

$$a = 20 \text{ mm/s}^2$$

12-5.

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

SOLUTION

$$s = t^2 - 6t + 5$$

$$v = \frac{ds}{dt} = 2t - 6$$

$$a = \frac{dv}{dt} = 2$$

$$v = 0 \text{ when } t = 3$$

$$s|_{t=0} = 5$$

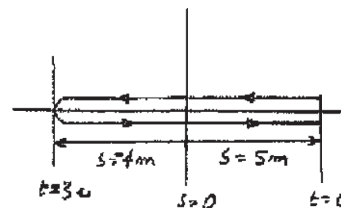
$$s|_{t=3} = -4$$

$$s|_{t=6} = 5$$

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{0}{6} = 0$$

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_T}{\Delta t} = \frac{9 + 9}{6} = 3 \text{ m/s}$$

$$a|_{t=6} = 2 \text{ m/s}^2$$

**Ans.****Ans.****Ans.****Ans:**

$$v_{\text{avg}} = 0$$

$$(v_{\text{sp}})_{\text{avg}} = 3 \text{ m/s}$$

$$a|_{t=6 \text{ s}} = 2 \text{ m/s}^2$$

12-6.

A stone A is dropped from rest down a well, and in 1 s another stone B is dropped from rest. Determine the distance between the stones another second later.

SOLUTION

$$+\downarrow s = s_1 + v_1 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(9.81)(2)^2$$

$$s_A = 19.62 \text{ m}$$

$$s_B = 0 + 0 + \frac{1}{2}(9.81)(1)^2$$

$$s_B = 4.91 \text{ m}$$

$$\Delta s = 19.62 - 4.91 = 14.71 \text{ m}$$

Ans.**Ans:**

$$\Delta s = 14.71 \text{ m}$$

12-7.

A bus starts from rest with a constant acceleration of 1 m/s^2 . Determine the time required for it to attain a speed of 25 m/s and the distance traveled.

SOLUTION***Kinematics:***

$v_0 = 0$, $v = 25 \text{ m/s}$, $s_0 = 0$, and $a_c = 1 \text{ m/s}^2$.

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$25 = 0 + (1)t$$

$$t = 25 \text{ s}$$

Ans.

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$25^2 = 0 + 2(1)(s - 0)$$

$$s = 312.5 \text{ m}$$

Ans.**Ans:**

$$t = 25 \text{ s}$$

$$s = 312.5 \text{ m}$$

***12-8.**

A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$$v = 12 - 3t^2$$

$$a = \frac{dv}{dt} = -6t|_{t=4} = -24 \text{ m/s}^2$$

$$\int_{-10}^s ds = \int_1^t v dt = \int_1^t (12 - 3t^2) dt$$

$$s + 10 = 12t - t^3 - 11$$

$$s = 12t - t^3 - 21$$

$$s|_{t=0} = -21$$

$$s|_{t=10} = -901$$

$$\Delta s = -901 - (-21) = -880 \text{ m}$$

From Eq. (1):

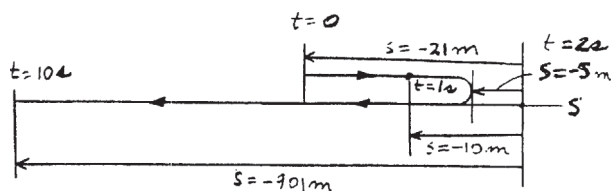
$$v = 0 \text{ when } t = 2 \text{ s}$$

$$s|_{t=2} = 12(2) - (2)^3 - 21 = -5$$

$$s_T = (21 - 5) + (901 - 5) = 912 \text{ m}$$

(1)

Ans.



Ans.

Ans.

Ans:

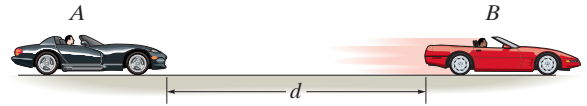
$$a = -24 \text{ m/s}^2$$

$$\Delta s = -880 \text{ m}$$

$$s_T = 912 \text{ m}$$

12-9.

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_A and v_B , respectively. If B maintains its constant speed, while A begins to decelerate at a_A , determine the distance d between the cars at the instant A stops.

**SOLUTION**

Motion of car A :

$$v = v_0 + a_c t$$

$$0 = v_A - a_A t \quad t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B :

$$s_B = v_B t = v_B \left(\frac{v_A}{a_A} \right) = \frac{v_A v_B}{a_A}$$

The distance between cars A and B is

$$s_{BA} = |s_B - s_A| = \left| \frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A} \right| = \left| \frac{2v_A v_B - v_A^2}{2a_A} \right|$$

Ans.

Ans:

$$s_{BA} = \left| \frac{2v_A v_B - v_A^2}{2a_A} \right|$$

12–10.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

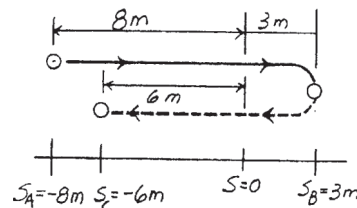
SOLUTION

Average Velocity: The displacement from A to C is $\Delta s = s_C - s_A = -6 - (-8) = 2$ m.

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{2}{4 + 5} = 0.222 \text{ m/s} \quad \text{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \rightarrow B} = 8 + 3 = 11.0$ m and $s_{B \rightarrow C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{\text{Tot}} = s_{A \rightarrow B} + s_{B \rightarrow C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{\text{sp}})_{\text{avg}} = \frac{s_{\text{Tot}}}{\Delta t} = \frac{20.0}{4 + 5} = 2.22 \text{ m/s} \quad \text{Ans.}$$

**Ans:**

$$v_{\text{avg}} = 0.222 \text{ m/s}$$

$$(v_{\text{sp}})_{\text{avg}} = 2.22 \text{ m/s}$$

12-11.

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$$v = v_1 + a_c t$$

$$120 = 70 + 6000(t)$$

$$t = 8.33(10^{-3}) \text{ hr} = 30 \text{ s}$$

Ans.

$$v^2 = v_1^2 + 2 a_c (s - s_1)$$

$$(120)^2 = 70^2 + 2(6000)(s - 0)$$

$$s = 0.792 \text{ km} = 792 \text{ m}$$

Ans.

Ans:
 $t = 30 \text{ s}$
 $s = 792 \text{ m}$

***12-12.**

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s², where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{(3s^{1/3} + s^{5/2})}$$

$$a \, ds = v \, dv$$

$$\int_1^2 \frac{5 \, ds}{(3s^{1/3} + s^{5/2})} = \int_0^v v \, dv$$

$$0.8351 = \frac{1}{2} v^2$$

$$v = 1.29 \text{ m/s}$$

Ans.

Ans:
 $v = 1.29 \text{ m/s}$

12–13.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1) \text{ m/s}^2$, where t is in seconds. If $s = 1 \text{ m}$ and $v = 2 \text{ m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6 \text{ s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION

$$a = 2t - 1$$

$$dv = a \, dt$$

$$\int_2^v dv = \int_0^t (2t - 1) dt$$

$$v = t^2 - t + 2$$

$$dx = v \, dt$$

$$\int_1^s ds = \int_0^t (t^2 - t + 2) dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

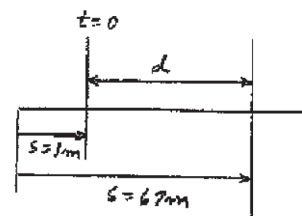
When $t = 6 \text{ s}$

$$v = 32 \text{ m/s}$$

$$s = 67 \text{ m}$$

Since $v \neq 0$ for $0 \leq t \leq 6 \text{ s}$, then

$$d = 67 - 1 = 66 \text{ m}$$



Ans.

Ans.

Ans.

Ans:

$$v = 32 \text{ m/s}$$

$$s = 67 \text{ m}$$

$$d = 66 \text{ m}$$

12–14.

A train starts from rest at station *A* and accelerates at 0.5 m/s^2 for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at 1 m/s^2 until it is brought to rest at station *B*. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60 \text{ s}$, and $a_c = 0.5 \text{ m/s}^2$. Thus,

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900 \text{ m}$$

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30 \text{ m/s}$$

For stage (2) motion, $v_0 = 30 \text{ m/s}$, $s_0 = 900 \text{ m}$, $a_c = 0$ and $t = 15(60) = 900 \text{ s}$. Thus,

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900 \text{ m}$$

For stage (3) motion, $v_0 = 30 \text{ m/s}$, $v = 0$, $s_0 = 27\,900 \text{ m}$ and $a_c = -1 \text{ m/s}^2$. Thus,

$$\left(\begin{array}{c} + \\ \rightarrow \end{array} \right) \quad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30 \text{ s}$$

$$\begin{array}{c} + \\ \rightarrow \end{array} \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350 \text{ m} = 28.4 \text{ km}$$

Ans.

Ans:
 $s = 28.4 \text{ km}$

12–15.

A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters. If $s = 2$ m when $t = 0$, determine the velocity and acceleration as functions of time.

SOLUTION

$$v = -4s^2$$

$$\frac{ds}{dt} = -4s^2$$

$$\int_2^s s^{-2} ds = \int_0^t -4 dt$$

$$-s^{-1} \Big|_2^s = -4t \Big|_0^t$$

$$t = \frac{1}{4}(s^{-1} - 0.5)$$

$$s = \frac{2}{8t + 1}$$

$$v = -4 \left(\frac{2}{8t + 1} \right)^2 = -\frac{16}{(8t + 1)^2} \text{ m/s}$$

Ans.

$$a = \frac{dv}{dt} = \frac{16(2)(8t + 1)(8)}{(8t + 1)^4} = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$

Ans.**Ans:**

$$v = \frac{16}{(8t + 1)^2} \text{ m/s}$$

$$a = \frac{256}{(8t + 1)^3} \text{ m/s}^2$$

***12-16.**

Determine the time required for a car to travel 1 km along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at 1.5 m/s^2 and decelerate at 2 m/s^2 .

SOLUTION

Using formulas of constant acceleration:

$$v_2 = 1.5 t_1$$

$$x = \frac{1}{2}(1.5)(t_1^2)$$

$$0 = v_2 - 2 t_2$$

$$1000 - x = v_2 t_2 - \frac{1}{2}(2)(t_2^2)$$

Combining equations:

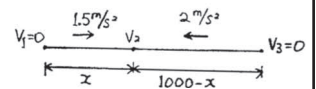
$$t_1 = 1.33 t_2; \quad v_2 = 2 t_2$$

$$x = 1.33 t_2^2$$

$$1000 - 1.33 t_2^2 = 2 t_2^2 - t_2^2$$

$$t_2 = 20.702 \text{ s}; \quad t_1 = 27.603 \text{ s}$$

$$t = t_1 + t_2 = 48.3 \text{ s}$$



Ans.

Ans:
 $t = 48.3 \text{ s}$

12–17.

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^v v^{-3} dv = \int_0^t -k dt$$

$$-\frac{1}{2}(v^{-2} - v_0^{-2}) = -kt$$

$$v = \left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{-\frac{1}{2}}$$

Ans.

$$ds = v dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \frac{2\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}}}{2k} \Big|_0^t$$

$$s = \frac{1}{k} \left[\left(2kt + \left(\frac{1}{v_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{v_0} \right]$$

Ans.**Ans:**

$$v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$$

$$s = \frac{1}{k} \left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0} \right]$$

12–18.

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})$ m/s², where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$\begin{aligned}
 ds &= \frac{v dv}{a} \\
 \int_0^s ds &= \int_{6 \text{ m/s}}^v \frac{v}{-1.5v^{1/2}} dv \\
 s &= \int_{6 \text{ m/s}}^v -0.6667 v^{1/2} dv \\
 &= \left(-0.4444v^{3/2} + 6.532 \right) \text{ m}
 \end{aligned}$$

When $v = 0$, $s = -0.4444 \left(0^{3/2} \right) + 6.532 = 6.53 \text{ m}$ **Ans.**

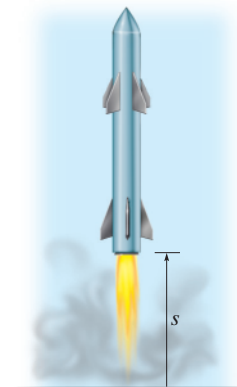
Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

$$\begin{aligned}
 dt &= \frac{dv}{a} \\
 \int_0^t dt &= - \int_{6 \text{ m/s}}^v \frac{dv}{1.5v^{1/2}} \\
 t &= -1.333 \left(v^{1/2} \right) \Big|_{6 \text{ m/s}}^v = \left(3.266 - 1.333v^{1/2} \right) \text{ s}
 \end{aligned}$$

When $v = 0$, $t = 3.266 - 1.333 \left(0^{1/2} \right) = 3.27 \text{ s}$ **Ans.**

Ans:
 $s = 6.53 \text{ m}$
 $t = 3.27 \text{ s}$

12–19. The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s) \text{ m/s}^2$, where s is in meters. Determine the rocket's velocity when $s = 2 \text{ km}$ and the time needed to reach this attitude. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$b = 6 \text{ m/s}^2 \quad c = 0.02 \text{ s}^{-2} \quad s_{pI} = 2000 \text{ m}$$

$$a_p = b + cs_p = v_p \frac{dv_p}{ds_p}$$

$$\int_0^{v_p} v_p dv_p = \int_0^{s_p} (b + cs_p) ds_p$$

$$\frac{v_p^2}{2} = bs_p + \frac{c}{2}s_p^2$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2bs_p + cs_p^2} \quad v_{pI} = \sqrt{2bs_{pI} + cs_{pI}^2} \quad v_{pI} = 322.49 \text{ m/s} \quad \text{Ans.}$$

$$t = \int_0^{s_p} \frac{1}{\sqrt{2bs_p + cs_p^2}} ds_p \quad t_I = \int_0^{s_{pI}} \frac{1}{\sqrt{2bs_p + cs_p^2}} ds_p \quad t_I = 19.27 \text{ s} \quad \text{Ans.}$$

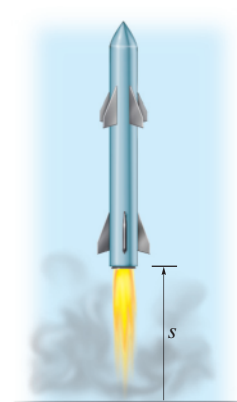
Ans:

$$v_{pI} = 322.49 \frac{\text{m}}{\text{s}}$$

$$t_I = 19.27 \text{ s}$$

***12–20.**

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s) \text{ m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100 \text{ m}$. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$a \, ds = v \, dv$$

$$\int_0^s (6 + 0.02s) \, ds = \int_0^v v \, dv$$

$$6s + 0.01s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12s + 0.02s^2}$$

$$ds = v \, dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12s + 0.02s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}} \ln \left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}} \right]_0^{100} = t$$

$$t = 5.62 \text{ s}$$

Ans.

Ans:
 $t = 5.62 \text{ s}$

12–21.

When a train is traveling along a straight track at 2 m/s, it begins to accelerate at $a = (60 v^{-4}) \text{ m/s}^2$, where v is in m/s. Determine its velocity v and the position 3 s after the acceleration.

**SOLUTION**

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60v^{-4}}$$

$$3 = \frac{1}{300} (v^5 - 32)$$

$$v = 3.925 \text{ m/s} = 3.93 \text{ m/s}$$

Ans.

$$ads = vdv$$

$$ds = \frac{v dv}{a} = \frac{1}{60} v^5 dv$$

$$\int_0^s ds = \frac{1}{60} \int_2^{3.925} v^5 dv$$

$$s = \frac{1}{60} \left(\frac{v^6}{6} \right) \bigg|_2^{3.925}$$

$$= 9.98 \text{ m}$$

Ans.**Ans:**

$$v = 3.93 \text{ m/s}$$

$$s = 9.98 \text{ m}$$

12-22.

The acceleration of a particle along a straight line is defined by $a = (2t - 9) \text{ m/s}^2$, where t is in seconds. At $t = 0$, $s = 1 \text{ m}$ and $v = 10 \text{ m/s}$. When $t = 9 \text{ s}$, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$$a = 2t - 9$$

$$\int_{10}^v dv = \int_0^t (2t - 9) dt$$

$$v - 10 = t^2 - 9t$$

$$v = t^2 - 9t + 10$$

$$\int_1^s ds = \int_0^t (t^2 - 9t + 10) dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5t^2 + 10t$$

$$s = \frac{1}{3}t^3 - 4.5t^2 + 10t + 1$$

Note when $v = t^2 - 9t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298 \text{ s}$, $s = 7.13 \text{ m}$

When $t = 7.701 \text{ s}$, $s = -36.63 \text{ m}$

When $t = 9 \text{ s}$, $s = -30.50 \text{ m}$

(a) $s = -30.5 \text{ m}$

Ans.

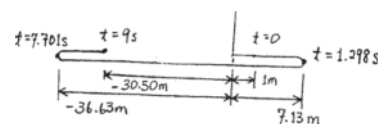
(b) $s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$$s_{Tot} = 56.0 \text{ m}$$

Ans.

(c) $v = 10 \text{ m/s}$

Ans.



Ans:

(a) $s = -30.5 \text{ m}$

(b) $s_{Tot} = 56.0 \text{ m}$

(c) $v = 10 \text{ m/s}$

12–23.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]$ m/s², where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \quad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81} \left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)} \right]$$

$$9.81t = 50 \ln \left(\frac{1 + 0.01v}{1 - 0.01v} \right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \quad (1)$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5 \text{ m/s} \quad \text{Ans.}$$

b) If $t \rightarrow \infty$, $\frac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{\max} = 100 \text{ m/s} \quad \text{Ans.}$$

Ans:

(a) $v = 45.5 \text{ m/s}$

(b) $v_{\max} = 100 \text{ m/s}$

***12-24.**

A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

SOLUTION

$$(+\downarrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$\begin{aligned} h &= 0 + (-6)(8) + \frac{1}{2} (9.81)(8)^2 \\ &= 265.92 \text{ m} \end{aligned}$$

During $t = 8$ s, the balloon rises

$$h' = vt = 6(8) = 48 \text{ m}$$

$$\text{Altitude} = h + h' = 265.92 + 48 = 314 \text{ m}$$

Ans.

$$(+\downarrow) \quad v = v_0 + a_c t$$

$$v = -6 + 9.81(8) = 72.5 \text{ m/s}$$

Ans.

Ans:
 $h = 314 \text{ m}$
 $v = 72.5 \text{ m/s}$

12–25.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v) \text{ m/s}^2$, where v is in meters per second. If $v = 20 \text{ m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^v \frac{dv}{v} = \int_0^t -2 dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = (20e^{-2t}) \text{ m/s}$$

Ans.

$$a = \frac{dv}{dt} = (-40e^{-2t}) \text{ m/s}^2$$

Ans.

$$\int_0^s ds = v dt = \int_0^t (20e^{-2t}) dt$$

$$s = -10e^{-2t} \Big|_0^t = -10(e^{-2t} - 1)$$

$$s = 10(1 - e^{-2t}) \text{ m}$$

Ans.**Ans:**

$$v = (20e^{-2t}) \text{ m/s}$$

$$a = (-40e^{-2t}) \text{ m/s}^2$$

$$s = 10(1 - e^{-2t}) \text{ m}$$