

Chapter 2

2.1 We wish to fabricate a planar waveguide in GaAs for light of wavelength $\lambda_0 = 1.1 \mu\text{m}$ that will operate in the single (fundamental) mode. If we assume a planar waveguide like that of Fig. 2.1 with the condition $n_2 - n_1 \gg n_2 - n_3$, what range of values can $n_2 - n_3$ have if $n_2 = 3.4$ and the thickness of the waveguiding layer $t = 3 \mu\text{m}$?

Solution. The cutoff condition is

$$\Delta n = n_2 - n_3 \leq \frac{(2M+1)^2}{32n_2} \left(\frac{\lambda_0}{t} \right)^2 \quad M = 0, 1, 2, \dots$$

Hence for fundamental mode ($M = 0$) propagation only we need

$$\begin{aligned} \frac{1}{32n_2} \left(\frac{\lambda_0}{t} \right)^2 &\leq n_2 - n_3 < \frac{9}{32n_2} \left(\frac{\lambda_0}{t} \right)^2 \\ \frac{1}{32 \times 3.4} \left(\frac{1.15}{3} \right)^2 &\leq n_2 - n_3 < \frac{9}{32 \times 3.4} \left(\frac{1.15}{3} \right)^2 \\ 0.00135 &\leq n_2 - n_3 \ll 0.0122. \end{aligned}$$

2.2 Repeat Problem 2.1 for the case $\lambda_0 = 1.06 \mu\text{m}$, all other parameters remaining unchanged.

Solution.

$$\begin{aligned} \frac{1}{32 \times 3.4} \left(\frac{1.06}{3} \right)^2 &\leq n_2 - n_3 < \frac{9}{32 \times 3.4} \left(\frac{1.06}{3} \right)^2 \\ 0.00115 &\leq n_2 - n_3 < 0.00103 \end{aligned}$$

2.3 Repeat Problems 2.1 and 2.2 for a waveguide of thickness $t = 6 \mu\text{m}$.

Solution.

$$\begin{aligned} \frac{1}{32 \times 3.4} \left(\frac{1.15}{6} \right)^2 &\leq n_2 - n_3 < \frac{9}{32 \times 3.4} \left(\frac{1.06}{6} \right)^2 \\ 0.000338 &\leq n_2 - n_3 < 0.00304 \\ \frac{1}{32 \times 3.4} \left(\frac{1.06}{6} \right)^2 &\leq n_2 - n_3 < \frac{9}{32 \times 3.4} \left(\frac{1.06}{6} \right)^2 \\ 0.000287 &\leq n_2 - n_3 < 0.00258 \end{aligned}$$

Note how small the required Δn are in all the cases calculated and also note the strong dependence on wavelength and on waveguide thickness.

2.4 In a planar waveguide like that of Fig. 2.8 with $n_2 = 2.0$, $n_3 = 1.6$, and $n_1 = 1$, what is the angle of propagation of the lowest order mode (θ_0) when cutoff occurs? Is this a maximum or a minimum angle for θ_0 ?

Solution. At cutoff we know

$$\cos \theta_m = \frac{n_3}{n_2}$$

or

$$\begin{aligned} \cos \theta_0 &= \frac{1.6}{2.0} \\ \theta_0 &= 36.87^\circ . \end{aligned}$$

We can tell that this is a maximum angle because

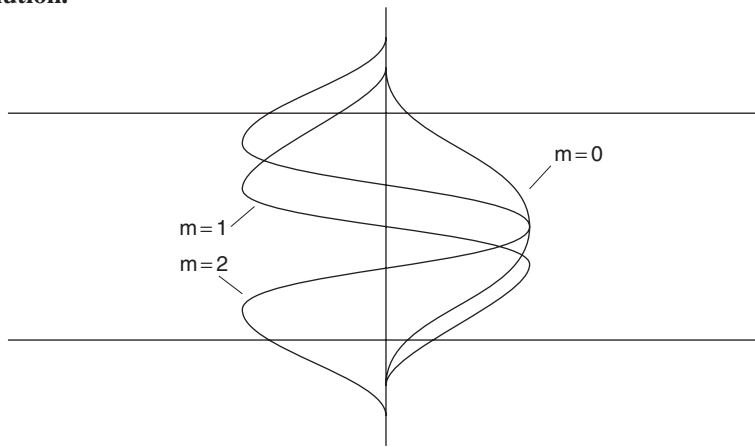
$$\cos \theta_m = \frac{\beta_m}{kn_2} ,$$

and from physical optics we know the condition required for waveguiding is

$$\beta_m \geq kn_3 .$$

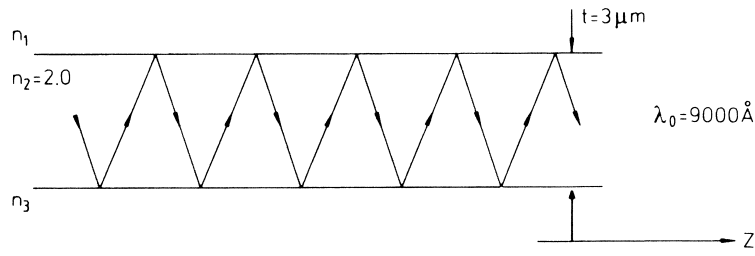
2.5 Sketch the three lowest order modes in a planar waveguide like that of Fig. 2.8 with $n_1 = n_3 < n_2$.

Solution.



2.6 A mode is propagating in a planar waveguide as shown with $\beta_m = 0.8 kn_2$. How many reflections at the $n_1 - n_2$ interface does the ray experience in traveling a distance of 1 cm in the z direction?

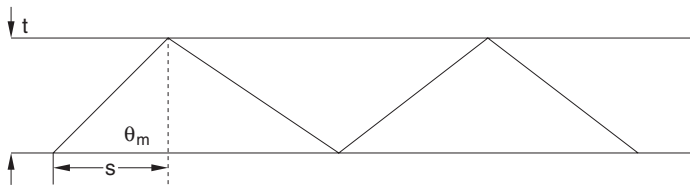
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Solution.

$$\frac{\beta_m}{kn_2} = 0.8 = \cos \theta_m$$

$$\theta_m = 36.87^\circ$$



from simple geometric considerations the number of bounces from each surface in length L is given by

$$\# \text{ of bounces} = \frac{L}{2s} = \frac{L}{2t \cot \theta_m}$$

for $L = 1 \text{ cm}$ and $t = 3 \mu\text{m}$

$$\# \text{ of bounces} = \frac{1}{2 \times 3 \times 10^{-4} \times \cot 36.87^\circ} = 1250 .$$

2.7 Show that the Goos–Hänchen phase shift goes to zero as the cutoff angle is approached for a waveguided optical mode.

Solution. At cutoff for the n_2 – n_3 interface,

$$\phi_2 = \theta_c = \sin^{-1}(n_3/n_2) .$$

The Goos–Hänchen shift for a TE wave is given by (2.1.21) as

$$\tan \phi_{23} = (n_2^2 \sin^2 \phi_2 - n_3^2)^{1/2} / (n_2 \cos \phi_2) ,$$

substituting $\phi_2 = \theta_c = \sin^{-1}(n_3/n_2)$

$$\tan \phi_{23} = (n_2^2 \cdot (n_3/n_2)^2 - n_3^2)^{1/2} / n_2 \cos(\sin^{-1}[n_3/n_2]) = 0 .$$

The same result for TM waves can be demonstrated by substituting into equation (2.1.22)

2.8 Calculate the Goos–Hänchen shifts for a TE mode guided with $\beta = 1.85k$ in a guide like that of Fig. 2.8, with $n_1 = 1.0$, $n_2 = 2.0$, $n_3 = 1.7$.

Solution.

$$\sin \phi_2 = \frac{\beta}{kn_2} = \frac{1.85k}{k \times 2} = 0.925$$

$$\therefore \phi_2 = 67.7^\circ$$

$$\begin{aligned} \tan \phi_{23} &= \frac{(n_2^2 \sin^2 \phi_2 - n_3^2)^{1/2}}{n_2 \cos \phi_2} \\ &= \frac{(4(0.925)^2 - (1.7)^2)^{1/2}}{2 \cos 67.7^\circ} \\ &= \frac{0.729}{2 \cos 67.7^\circ} = 0.961 \\ \phi_{23} &= 43.9^\circ \end{aligned}$$

$$\begin{aligned} \tan \phi_{21} &= \frac{(n_2^2 \sin^2 \phi_2 - n_1^2)^{1/2}}{n_2 \cos \phi_2} \\ &= \frac{(4(0.925)^2 - 1)^{1/2}}{2 \cos 67.7^\circ} \\ &= \frac{1.556}{2 \cos 67.7^\circ} = 2.05 \\ \phi_{21} &= 64.0^\circ \end{aligned}$$

The Goos–Hänchen Shifts are

$$\begin{aligned} -2\phi_{23} &= -87.8^\circ \\ -2\phi_{21} &= -128^\circ . \end{aligned}$$

2.9 Show by drawing the vectorial relationship between the propagation constants (as in Fig. 2.9) How β , kn_2 and h change in relative magnitude and angle as one goes from the lowest-order mode in a waveguide progressively to higher-order modes.

Solution.

$$kn_2 = \text{constant}$$

$$\text{as mode } \uparrow, \theta_m \uparrow$$

$$\cos \theta_m \downarrow$$

$$\beta_m = kn_2 \cos \theta_m$$

$$\therefore \text{ as mode } \uparrow, \beta_m \downarrow$$

$$h = (n_2^2 k^2 - \beta^2)^{1/2}$$

$$\therefore \text{ as mode } \uparrow, h \uparrow$$

See below

