

Chapter 2: STATICS—A REVIEW

Solution 2.1

The free-body diagram of the disk is shown in Figure S2.1.

Note: For force balance (equilibrium) the reaction R must pass through O since the other two forces (T and mg) intersect at O . Hence, even though we did not neglect friction, the frictional force at B is zero for this system.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: T \sin \theta - R = 0 \Rightarrow R = T \sin \theta \quad (i)$$

$$\uparrow \sum F_y = 0: T \cos \theta - mg = 0 \Rightarrow T = \frac{mg}{\cos \theta} \quad (ii)$$

Note: The moment balance is satisfied in the given configuration. Hence, no new equation is generated through this condition.

Substitute (ii) in (1): $R = mg \tan \theta \quad (iii)$

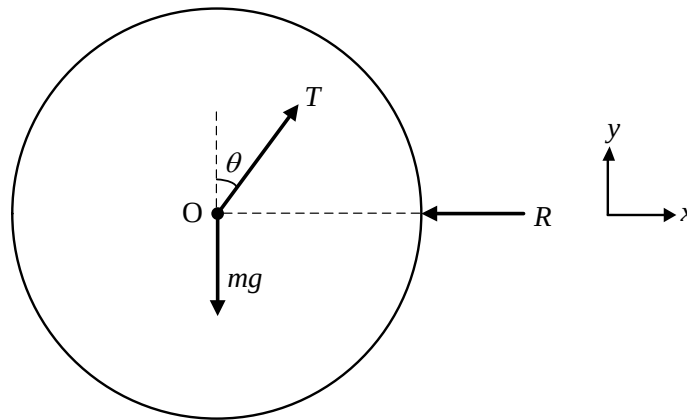


Figure S2.1: Free-body diagram of the disk.

From geometry (right-angled triangle AOB):

$$\sin \theta = \frac{r}{L} = \frac{300}{500} = 0.6; \cos \theta = \frac{400}{500} = 0.8; \tan \theta = \frac{300}{400} = 0.75;$$

Substitute numerical values.

$$(ii): T = \frac{20 \times 9.81}{0.8} = 245 \text{ N}$$

$$(iii): R = 20 \times 9.81 \times 0.75 = 147 \text{ N}$$

Solution 2.2

The forces acting at point C of the traffic light are shown in Figure S2.2.

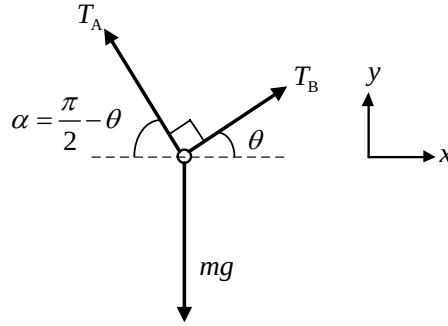


Figure S2.2: Forces at point C of the traffic light.

From geometry, ACB is a right-angled triangle with $\alpha = \frac{\pi}{2} - \theta$.

Hence,

$$\sin \theta = \frac{3}{5} = 0.6; \cos \theta = \frac{4}{5} = 0.8; \tan \theta = \frac{3}{4} = 0.75$$

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: -T_A \sin \theta + T_B \cos \theta = 0 \Rightarrow -T_A \tan \theta + T_B = 0 \quad (i)$$

$$\uparrow \sum F_y = 0: T_A \cos \theta + T_B \sin \theta - mg = 0 \Rightarrow \frac{T_A}{\tan \theta} + T_B = \frac{mg}{\sin \theta} \quad (ii)$$

$$(ii) - (i): T_A \left(\tan \theta + \frac{1}{\tan \theta} \right) = \frac{mg}{\sin \theta} \quad (iii)$$

Substitute numerical values:

$$(iii): T_A \left(\frac{1}{0.75} + 0.75 \right) = \frac{50 \times 9.81}{0.6} \Rightarrow T_A = 392 \text{ N}$$

$$(i): T_B = T_A \tan \theta = 392 \times 0.75 = 294 \text{ N}$$

Solution 2.3

Free-body diagram of the disk with the handle is shown in Figure S2.3.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: -R_1 + F + P \sin \theta = 0 \quad (i)$$

$$\uparrow \sum F_y = 0: R - mg - P \cos \theta = 0 \Rightarrow R = mg + P \cos \theta \quad (ii)$$

where,

R_1 = reaction at B (normal because frictionless)

R = normal reaction at A

F = frictional resistance force at A

$$\curvearrowleft \sum M_O = 0: F \times r - P \times L = 0 \Rightarrow F = \frac{L}{r} P \quad (iii)$$

Substitute (iii) in (i):

$$R_1 = F + P \sin \theta = \frac{L}{r} P + P \sin \theta \quad (iv)$$

Substitute numerical values:

$$(iii): F = \frac{1.0}{0.5} \times 200 = 400.0 \text{ N}$$

$$(ii): R = 50 \times 9.81 + 200 \cos 30^\circ = 663.7 \text{ N}$$

$$(iv): R_1 = \frac{1.0}{0.5} \times 200 + 200 \sin 30^\circ = 500.0 \text{ N}$$

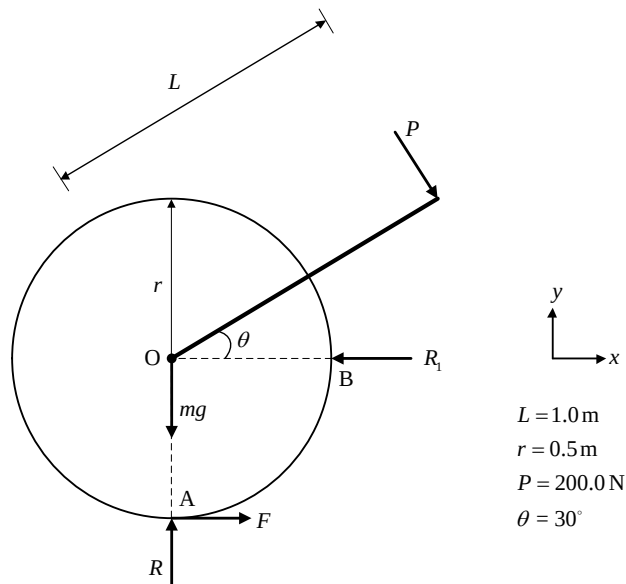


Figure S2.3: Free-body diagram of disk with handle.

Solution 2.4

Free-body diagram of the shaft with the pulley is shown in Figure S2.4.

Note: Ball bearings do not exert moments or axial forces on the shaft.

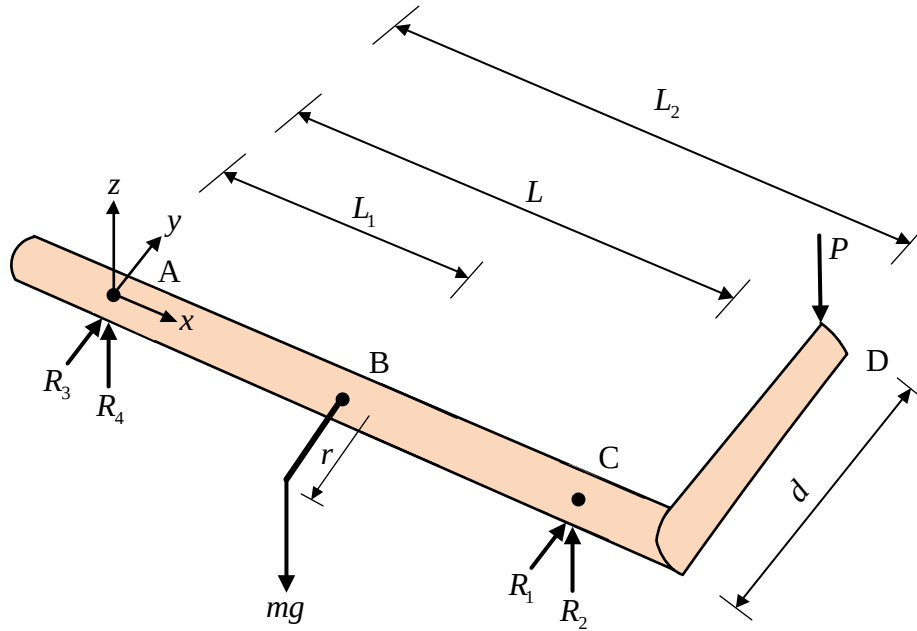


Figure S2.4 Free-body diagram of the shaft with pulley.

Equilibrium Equations:

Note the Cartesian coordinate system (x, y, z) located at A.

$$\sum M_x = 0: mg \times r - P \times d = 0 \Rightarrow P = \frac{r}{d} mg = \frac{0.2}{0.5} \times 20 \times 9.81 = 78.5 \text{ N}$$

$$\sum M_z = 0: R_1 \times L = 0 \Rightarrow R_1 = 0$$

$$\sum F_y = 0: R_3 + R_1 = 0 \Rightarrow R_3 = 0$$

$$\begin{aligned} \sum M_y = 0: mg \times L_1 - R_2 \times L + P \times L_2 &= 0 \\ \Rightarrow R_2 &= \frac{mgL_1 + PL_2}{L} = \frac{20 \times 9.81 \times 0.6 + 78.5 \times 1.2}{1.0} \text{ N} = 211.9 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum F_z = 0: R_4 - mg + R_2 - P &= 0 \\ \Rightarrow R_4 &= mg + P - R_2 = 20 \times 9.81 + 78.5 - 211.9 = 62.8 \text{ N} \end{aligned}$$

Solution 2.5

The free-body diagram of the structure is shown in Figure S2.5. Note the directions of x and y as given by the Cartesian coordinate frame.

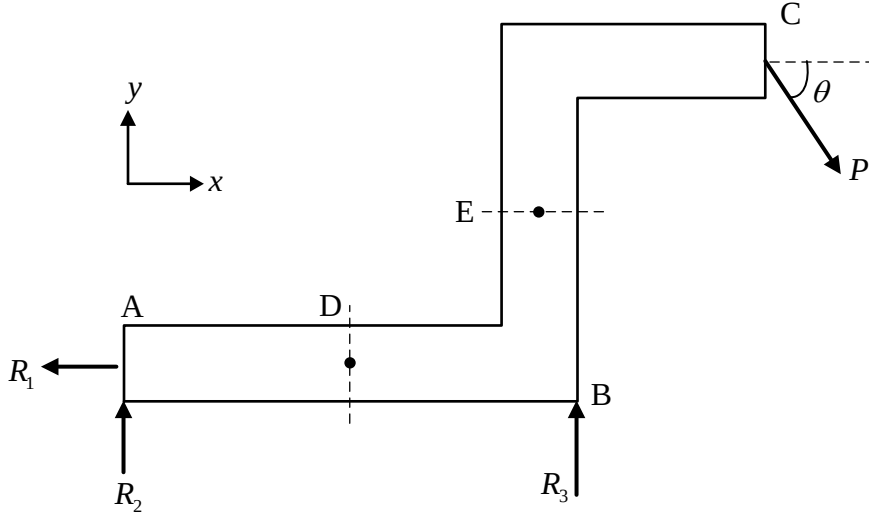


Figure S2.5: Free-body diagram of the structure.

Note: A smooth roller support cannot exert a lateral force on the structure

R_1 = horizontal component of the reaction at A

R_2 = vertical component of the reaction at A

R_3 = reaction at B (vertical, because of smooth roller)

(a)

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: -R_1 + P \cos \theta = 0 \Rightarrow R_1 = P \cos \theta = 200 \times \cos 60^\circ = 100 \text{ N}$$

$$\uparrow \sum F_y = 0: R_2 + R_3 - P \sin \theta = 0 \Rightarrow R_2 + R_3 = P \sin \theta \quad (\text{i})$$

$$\curvearrowleft \sum M_A = 0: R_3 \times L - P \cos \theta \times h - P \sin \theta \times (L + \frac{L}{2}) = 0$$

$$\Rightarrow R_3 = \frac{h}{L} P \cos \theta + 1.5 P \sin \theta = \frac{0.8}{1.0} \times 200 \times \frac{1}{2} + 1.5 \times 200 \times \frac{\sqrt{3}}{2} \text{ N} = 340 \text{ N}$$

Substitute in (i):

$$R_2 = P \sin \theta - R_3 = P \sin \theta - (\frac{h}{L} P \cos \theta + 1.5 P \sin \theta) = -0.5 P \sin \theta - \frac{h}{L} P \cos \theta \quad (\text{ii})$$

$$\Rightarrow R_2 = -0.5 \times 200 \times \frac{\sqrt{3}}{2} - \frac{0.8}{1.0} \times 200 \times \frac{1}{2} = -167 \text{ N}$$

(b)

Make a virtual X-section at D and consider the resulting free body AD. Its free-body diagram is shown in Figure S2.5 (a).

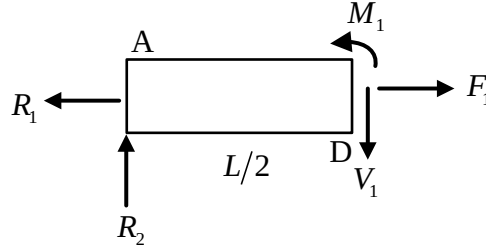


Figure S2.5(a): Free-body diagram of AD.

Note: The shear force V_1 and the bending moment M_1 at the X-section D are marked in their +ve directions by convention. F_1 is marked as +ve when in tension.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: -R_1 + F_1 = 0 \Rightarrow F_1 = R_1 = P \cos \theta = 100 \text{ N}$$

$$\uparrow \sum F_y = 0: R_2 - V_1 = 0 \Rightarrow V_1 = R_2$$

(ii):

$$V_1 = -0.5P \sin \theta - \frac{h}{L} P \cos \theta = -167 \text{ N}$$

$$\curvearrowleft \sum M_D = 0: M_1 - R_2 \times \frac{L}{2} = 0 \Rightarrow M_1 = \frac{R_2 L}{2}$$

$$(ii) \Rightarrow M_1 = (-0.25P \sin \theta - \frac{h}{2L} P \cos \theta) \times L = -167 \times \frac{1.0}{2} \text{ N} \cdot \text{m} = -83.5 \text{ N} \cdot \text{m}$$

Next, make a virtual X-section at E and consider the resulting free body EC. Its free-body diagram is shown in Figure S2.5(b).

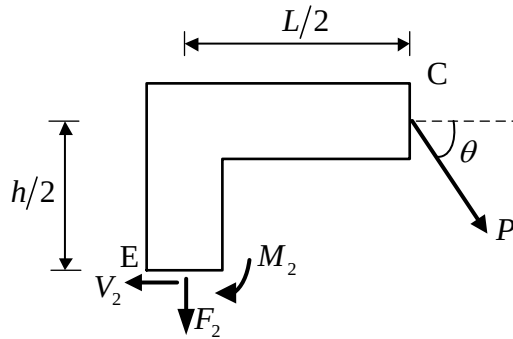


Figure S2.5(b): Free-body diagram of EC.

Note: The shear force V_2 and the bending moment M_2 at E are marked in their +ve directions, by convention. F_2 is marked as +ve when in tension.

Equilibrium Equations:

$$\rightarrow \sum F_x = 0: -V_2 + P \cos \theta = 0 \Rightarrow V_2 = P \cos \theta = 200 \times \frac{1}{2} = 100 \text{ N}$$

$$\uparrow \sum F_y = 0: -F_2 - P \sin \theta = 0 \Rightarrow F_2 = -P \sin \theta = -200 \times \frac{\sqrt{3}}{2} = -173 \text{ N}$$

(i.e., it is in compression)

$$\curvearrowleft \sum M_E = 0: -M_2 - P \cos \theta \times \frac{h}{2} - P \sin \theta \times \frac{L}{2} = 0$$

$$\Rightarrow M_2 = -\frac{P}{2}(h \cos \theta + L \sin \theta) = -\frac{200}{2} \times (0.8 \times \frac{1}{2} + 1.0 \times \frac{\sqrt{3}}{2}) \text{ N} \cdot \text{m} = -127 \text{ N} \cdot \text{m}$$

Solution 2.6

It is seen that both members are two-force members. Assume that the members are in tension (by convention).

From geometry: $\cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$; $\sin \theta = \frac{\sqrt{3}}{2}$

Case (a):

Consider Joint C (Figure S2.6(a)):

$$\rightarrow \sum F_x = 0 \Rightarrow -F_{BC} - F_{AC} \cos \theta = 0$$

$$\Rightarrow F_{BC} = -\frac{1}{2} F_{AC}$$

$$\uparrow \sum F_y = 0 \Rightarrow F_{AC} \sin \theta - P = 0$$

$$\Rightarrow F_{AC} = \frac{P}{\sin \theta} = \frac{20}{\sqrt{3}/2} = \frac{40}{\sqrt{3}} \text{ kN} \quad (\text{tension})$$

$$\Rightarrow F_{BC} = -\frac{20}{\sqrt{3}} \text{ kN} \quad (\text{compression})$$

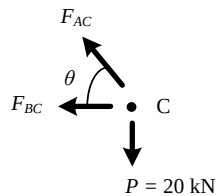


Figure S2.6(a): Forces at joint C.

Consider Joint B (Figure S2.6(b)):

$$\rightarrow \sum F_x = 0 \Rightarrow -R_B + F_{BC} = 0 \Rightarrow R_B = F_{BC} = -\frac{20}{\sqrt{3}} \text{ kN}$$

(i.e., acting to the right)

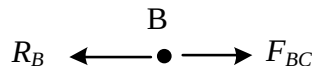


Figure S2.6(b): Forces at joint B.

Consider Joint A (Figure S2.6(c)):

$$R_A = F_{AC} = \frac{40}{\sqrt{3}} \text{ kN}$$

(acting upward at 60° from left horizontal)

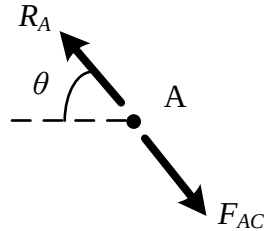


Figure S2.6(c): Forces at joint A.

Case (b):

Joint C (Figure S2.6(d)):

Clearly (by inspection; No need to write equations)

$$F_{AC} = 0$$

$$F_{BC} = P = 20 \text{ kN} \quad (\text{i.e., in tension})$$

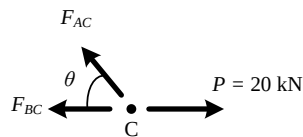


Figure S2.6(d): Forces at joint C.

Joint B(Figure S2.6(e)):

$$R_B = F_{BC} = 20 \text{ kN} \quad (\text{i.e., acting to the left})$$

Also,

$$R_A = 0$$

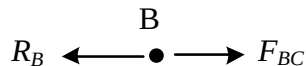


Figure S2.6(d): Forces at joint B.

Solution 2.7

$$\text{Load } P = 200g = 200 \times 9.81 \text{ N} = 1962.0 \text{ N}$$

$$\text{From geometry: } \theta = 22.5^\circ; \quad AC = CD = \sqrt{2} \text{ m}$$

Free-body diagram of the structure of interest is shown in Figure S2.7(a).
There are no moments at A and B because they are pin joints (frictionless).

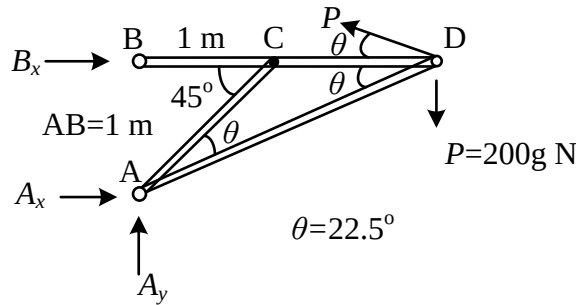


Figure S2.7(a): Free-body diagram of the boom.

Note: BC is a two-force member $\Rightarrow$ Reaction at B should be along BC, given by B_x . This will be further confirmed below, by analyzing joint B.

Equilibrium Equations for Boom:

$$\rightarrow \sum F_x = 0 \Rightarrow A_x + B_x - P \cos \theta = 0 \quad (i)$$

$$\uparrow \sum F_y = 0 \Rightarrow A_y - P + P \sin \theta = 0 \quad (ii)$$

$$\curvearrowright \sum M_A = 0 \Rightarrow -B_x \times 1 - P \times (1 + \sqrt{2}) + P \sin \theta \times (1 + \sqrt{2}) + P \cos \theta \times 1 = 0 \quad (iii)$$

We get:

$$(ii): \quad A_y = P(1 - \sin \theta) = 1962.0(1 - \sin 22.5^\circ) = 1211.2 \text{ N}$$

$$(iii): \quad B_x = -P(1 - \sin \theta)(1 + \sqrt{2}) = -1962.0(1 - \sin 22.5^\circ)(1 + \sqrt{2}) + 1962 \cos 22.5^\circ = -1111.1 \text{ N}$$

$$(i): \quad A_x = P \cos \theta - B_x = 1962.0 \cos 22.5^\circ + 1111.1 \text{ N} = 2923.7 \text{ N}$$

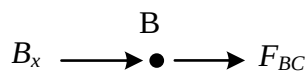
Note: Instead of writing (iii) above, we could have taken moments about B, and obtained an equation for A_x . Then (i) would give B_x .

Method of Joints:

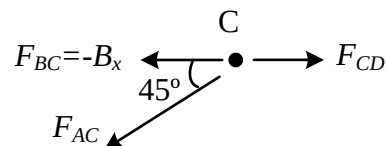
To determine the loads in the remaining rods of the truss, we use the method of joints.

Sign Convention: Rods are in tension.

(b)



(c)



(d)

(e)



Figure S2.7: Forces at: (b) joint B; (c) joint C; (d) joint C; (e) joint A.

Joint B (Figure S2.7(b)):

$$\rightarrow \sum F_x = 0 \Rightarrow B_x + F_{BC} = 0 \Rightarrow$$

$$\text{Load in BC} = F_{BC} = -B_x = 2924.0 \text{ N (tension)}$$

Joint C (Figure S2.7(c)):

$$\uparrow \sum F_y = 0 \Rightarrow -F_{AC} \sin 45^\circ = 0 \Rightarrow F_{AC} = 0$$

$$\rightarrow \sum F_x = 0 \Rightarrow -F_{BC} + F_{CD} = 0 \Rightarrow F_{CD} = F_{BC} = 2924.0 \text{ N}$$

Joint D (Figure S2.7(d)):

$$\uparrow \sum F_y = 0 \Rightarrow -F_{AD} \sin \theta + P \sin \theta - P = 0$$

$$\Rightarrow F_{AD} = -\frac{P(1 - \sin \theta)}{\sin \theta} = -\frac{1962.0 \times (1 - \sin 22.5^\circ)}{\sin 22.5^\circ} = -3165.0 \text{ N}$$

Note: This means, the member AD is in compression (which should be intuitive).

The same result may be obtained by considering Joint A.

Joint A (Figure S2.7(e)):

$$\uparrow \sum F_y = 0 \Rightarrow A_y + F_{AD} \sin \theta = 0$$

$$\Rightarrow F_{AD} = -\frac{(1 - \sin \theta)P}{\sin \theta} = -3165.0 \text{ N}$$

Solution 2.8

Free-body diagrams of the components of the pipe gripper are shown in Figure S2.8.

(a)