

Chapter 1

- 1.1 Since $pV = mRT$, $\frac{V_1}{V_1} = \frac{T_1 p_2}{T_2 p_1}$
 $\therefore V_1 = \frac{\pi}{6} (20 \text{ m})^3 \frac{288.15}{233.15} \frac{1.1}{101.3} = 56.2 \text{ m}^3$
- 1.2 $\rho = \frac{p}{RT} = \frac{1.4 \times 10^5 \text{ N} \cdot \text{m}^{-2}}{287 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} \times 323.15 \text{ K}} = 1.51 \text{ kg} \cdot \text{m}^{-3}$
- 1.3 $K = \rho \frac{\partial p}{\partial \rho}$ Assume K constant. Then $\ln(\rho/\rho_0) = \frac{p - p_0}{K}$
 $\therefore \rho = \rho_0 \exp\left(\frac{p - p_0}{K}\right) = 1025 \text{ kg} \cdot \text{m}^{-3} \exp\left(\frac{81.7 \times 10^6}{2.34 \times 10^9}\right)$
 $= 1061 \text{ kg} \cdot \text{m}^{-3}$
- 1.4 $\rho = \frac{\mu}{\nu} = \frac{2 \times 10^{-5} \text{ N} \cdot \text{s} \cdot \text{m}^{-2}}{15 \times 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}} = 1.333 \text{ kg} \cdot \text{m}^{-3}$
 $R = \frac{p}{\rho T} = \frac{1.013 \times 10^5 \text{ N} \cdot \text{m}^{-2}}{1.333 \text{ kg} \cdot \text{m}^{-3} \times 293.15 \text{ K}} = 259.2 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
 $\therefore M = \frac{8310}{259.2} = 32.06$
- 1.5 $\mu = \nu \rho = 400 \times 10^{-6} \text{ m}^2 \cdot \text{s}^{-1} \times 850 \text{ kg} \cdot \text{m}^{-3} = 0.34 \text{ Pa} \cdot \text{s}$
 Velocity gradient $= \frac{0.12 \text{ m} \cdot \text{s}^{-1}}{0.1 \times 10^{-3} \text{ m}} = 1200 \text{ s}^{-1}$
 Area $= \pi 0.2 \times 1.2 \text{ m}^2 = 0.754 \text{ m}^2$
 Force $= 0.754 \text{ m}^2 \times 0.34 \text{ Pa} \cdot \text{s} \times 1200 \text{ s}^{-1} = 307.6 \text{ N}$

$$\begin{aligned}
1.6 \quad \text{Total force on plate} &= \text{Area} \times \mu \left\{ \left(\frac{\partial u}{\partial y} \right)_{\text{side A}} + \left(\frac{\partial u}{\partial y} \right)_{\text{side B}} \right\} \\
&= (0.25 \text{ m})^2 \times 0.7 \text{ Pa} \cdot \text{s} \left\{ \frac{0.15 \text{ m} \cdot \text{s}^{-1}}{0.006 \text{ m}} + \frac{0.15 \text{ m} \cdot \text{s}^{-1}}{0.019 \text{ m}} \right\} \\
&= \mathbf{1.439 \text{ N}}
\end{aligned}$$

$$1.7 \quad \text{For annulus, radius } r, \text{ width } \delta r$$

$$\begin{aligned}
\text{Force} &= \text{Area} \times \mu \times \frac{\text{Velocity}}{\text{Clearance}} = 2\pi r \delta r \mu \frac{\omega r}{c} \\
\therefore \text{Torque} &= \text{Force} \times r = 2\pi r^3 \delta r \frac{\mu \omega}{c} \\
\text{Total torque} &= \int_0^R 2\pi r^3 \frac{\mu \omega}{c} dr = \frac{\pi R^4 \mu \omega}{2c} \\
&= \frac{\pi (0.1 \text{ m})^4 0.14 \text{ Pa} \cdot \text{s} \times 2\pi \times 7 \text{ rad} \cdot \text{s}^{-1}}{2 \times 0.00013 \text{ m}} = \mathbf{7.44 \text{ N} \cdot \text{m}}
\end{aligned}$$

$$1.8 \quad p = \frac{2\gamma}{d} = \frac{2 \times 0.073 \text{ N} \cdot \text{m}^{-1}}{0.004 \text{ m}} = \mathbf{36.5 \text{ Pa}}$$

$$\begin{aligned}
1.9 \quad h &= \frac{4\gamma \cos \theta}{\rho g d} = \frac{4 \times 0.073 \text{ N} \cdot \text{m}^{-1} \times 1}{1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1} \times 0.005 \text{ m}} \\
&= 0.00595 \text{ m} = \mathbf{5.95 \text{ mm}}
\end{aligned}$$

$$\begin{aligned}
1.10 \quad h &= \frac{4 \times 0.377 \text{ N} \cdot \text{m}^{-1} \times \cos 140^\circ}{(13.56 - 1) 1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1} \times 0.006 \text{ m}} \\
&= \mathbf{-1.563 \text{ mm}}
\end{aligned}$$

$$\begin{aligned}
1.11 \quad Re &= \frac{u d \rho}{\mu} = \frac{4 Q \rho}{\pi d \mu} = \frac{4 \times 0.0025 \text{ m}^3 \cdot \text{s}^{-1} \times 900 \text{ kg} \cdot \text{m}^{-3}}{\pi 0.05 \text{ m} \times 0.038 \text{ N} \cdot \text{s} \cdot \text{m}^{-2}} = \mathbf{1508} \\
u &= \frac{2000 \mu}{d \rho} = \frac{2000 \times 0.038 \text{ N} \cdot \text{s} \cdot \text{m}^{-2}}{0.05 \text{ m} \times 900 \text{ kg} \cdot \text{m}^{-3}} = \mathbf{1.689 \text{ m} \cdot \text{s}^{-1}}
\end{aligned}$$

$$1.12 \quad Re = \frac{4 Q \rho}{\pi d \mu} = \frac{4 \times 0.01 \text{ m}^3 \cdot \text{s}^{-1}}{\pi 0.08 \text{ m} \times 370 \times 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}} = \mathbf{430} \therefore \text{Laminar}$$

Chapter 2

$$2.1 \quad h = \frac{p}{\rho g} = \frac{200 \times 10^3 \text{ N} \cdot \text{m}^{-2}}{1590 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1}} = \mathbf{12.82 \text{ m}}$$

2.2 Pressure depends only on depth below free surface.

$$(a) \quad p = \rho g h = (820 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1})(3 - 0.15) \text{ m} \\ = 22\,930 \text{ N} \cdot \text{m}^{-2} = \mathbf{22.93 \text{ kPa}}$$

$$(b) \quad p = 820 \times 9.81 \text{ N} \cdot \text{m}^{-3} \times (3 + 2) \text{ m} = \mathbf{40.2 \text{ kPa}}$$

$$(c) \quad p = 820 \times 9.81 \text{ N} \cdot \text{m}^{-3} \times \{3 + 2 - (1.2 \sin 30^\circ + 0.6)\} \text{ m} \\ = 820 \times 9.81 \times 3.8 \text{ N} \cdot \text{m}^{-2} = \mathbf{30.57 \text{ kPa}}$$

$$(d) \quad \text{Load} = \text{Pressure} \times \text{Area} \\ = 820 \times 9.81 \times 3 \text{ N} \cdot \text{m}^{-2} \times (3.5 \times 2.5) \text{ m}^2 = \mathbf{211.2 \text{ kN}}$$

$$2.3 \quad h_{\text{air}} = \frac{p}{\rho_{\text{air}} g} = \frac{\rho_{\text{water}} g h_{\text{water}}}{\rho_{\text{air}} g} = \frac{\rho_{\text{water}}}{\rho_{\text{air}}} h_{\text{water}} \\ = \frac{1000 \text{ kg} \cdot \text{m}^{-3} \times 287 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1} \times 288.15 \text{ K}}{1.013 \times 10^5 \text{ N} \cdot \text{m}^{-2}} 0.075 \text{ m} \\ = \mathbf{61.2 \text{ m}}$$

$$2.4 \quad pV = \text{constant}$$

$$\therefore \left(\frac{d}{4 \text{ mm}} \right)^3 = \frac{101.3 \times 10^3 \text{ Pa} + 1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1} \times 9 \text{ m}}{101.3 \times 10^3 \text{ Pa}}$$

whence $d = \mathbf{4.93 \text{ mm}}$

$$2.5 \quad \Delta p = 820 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1} \times 2 \text{ m} + (13.56 - 0.82) \\ \times 1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1} \times 0.225 \text{ m} = \mathbf{44.2 \text{ kPa}}$$

$$\frac{\Delta p^*}{\rho g} = \Delta h = \frac{0.225 \text{ m}(13.56 - 0.82)1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1}}{820 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1}}$$

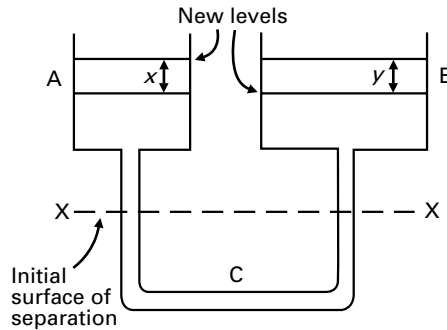
$$= 3.496 \text{ m}$$

$$44\,200 \text{ N} \cdot \text{m}^{-2} = 820 \times 9.81 \times 2 \text{ N} \cdot \text{m}^{-2} + x(0.82 - 0.74)1000$$

$$\times 9.81 \text{ N} \cdot \text{m}^{-3}$$

$$\text{whence } x = 35.83 \text{ m}$$

2.6



Movement of fluid in

$$C = 60 \text{ mm} \times 70 \text{ mm}^2$$

$$= (500 \text{ mm}^2)x$$

$$= (800 \text{ mm}^2)y$$

$$\therefore x = 8.4 \text{ mm};$$

$$y = 5.25 \text{ mm}$$

Measuring above XX: Initially $0.8h_A = 0.9h_B$ Later: $800 \times 9.81(\text{Old } h_A - 60 + 8.4)10^{-3} \text{ Pa}$

$$= p + 900 \times 9.81(\text{Old } h_B - 60 + 5.25)10^{-3} \text{ Pa}$$

$$\therefore p = 9.81 \times 10^{-3}(-800 \times 51.6 + 900 \times 65.25) \text{ Pa} = 171.1 \text{ Pa}$$

$$2.7 \quad \text{From eqn 2.7} \quad p = p_0 \left(1 - \frac{\lambda z}{T_0}\right)^{g/R\lambda}$$

$$= 101.5 \text{ Pa} \left(1 - \frac{0.0065 \times 7500}{288.15}\right)^{9.81/287 \times 0.0065}$$

$$= 38.3 \text{ kPa}$$

$$2.8 \quad \frac{p}{p_0} = \left(\frac{T_0 - \lambda z}{T_0}\right)^{g/R\lambda} = \left(\frac{T_{\text{top}}}{T_{\text{top}} + \lambda z}\right)^{g/R\lambda}$$

$$\therefore z = \frac{T_{\text{top}}}{\lambda} \left\{ \left(\frac{p_0}{p}\right)^{R\lambda/g} - 1 \right\}$$

$$= \frac{268.15}{0.0065} \text{ m} \left\{ \left(\frac{749}{566}\right)^{287 \times 0.0065/9.81} - 1 \right\}$$

$$= 2257 \text{ m}$$

$$2.9 \quad F = (1.2 \times 1.8) \text{ m}^2 \times 1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1} \\ \times (x + 0.9 \sin 30^\circ) \text{ m}$$

- (a) $2160 \times 9.81 \text{ N} \cdot \text{m}^{-1} \times 0.45 \text{ m} = \mathbf{9.54 \text{ kN}}$
 (b) $2160 \times 9.81 \text{ N} \cdot \text{m}^{-1} \times 0.95 \text{ m} = \mathbf{20.13 \text{ kN}}$
 (c) $2160 \times 9.81 \text{ N} \cdot \text{m}^{-1} \times 30.45 \text{ m} = \mathbf{645 \text{ kN}}$

Centre of pressure is at slant depth $\frac{(bd^3/12) + bd(2x + 0.9)^2}{bd(2x + 0.9)}$

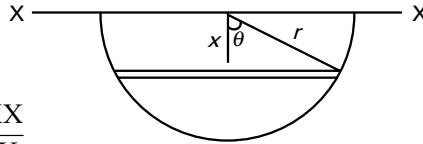
$$= \frac{d^2}{12(2x + 0.9)} + 2x + 0.9 (\text{metres})$$

$$= \frac{(1.8 \text{ m})^2}{12(2x + 0.9)} + 2x + 0.9 \text{ m}$$

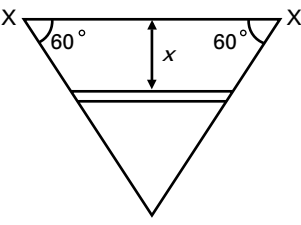
that is $\left\{ \frac{1.8^2}{12(2x + 0.9)} + 0.9 \right\} \text{ m from upper edge}$

= (a) **1.2 m**; (b) **1.042 m**; (c) **0.904 m** from upper edge

- 2.10 By symmetry, centre of pressure is on vertical centre-line



$$\begin{aligned} \text{Depth} &= \frac{\text{2nd moment about XX}}{\text{1st moment about XX}} \\ &= \frac{\int_0^r x^2 2(r^2 - x^2)^{1/2} dx}{\int_0^r x 2(r^2 - x^2)^{1/2} dx} \\ &= \frac{\int_{\pi/2}^0 (r \cos \theta)^2 2r \sin \theta (-r \sin \theta d\theta)}{\int_{\pi/2}^0 r \cos \theta 2r \sin \theta (-r \sin \theta d\theta)} \\ &= \frac{r \int_0^{\pi/2} \cos^2 \theta \sin^2 \theta d\theta}{\int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta} \\ &= \frac{r \int_0^{\pi} \frac{1}{8} \sin^2 2\theta d(2\theta)}{\left[\frac{1}{3} \sin^3 \theta \right]_0^{\pi/2}} \\ &= \frac{r/8 [2\theta/2 - (1/4) \sin 4\theta]_0^{2\theta=\pi}}{1/3} \\ &= \frac{3}{8} r \frac{\pi}{2} = \frac{3\pi d}{32} \end{aligned}$$

2.11 

Full depth = $(2.5 \text{ m}) \sin 60^\circ$

Breadth of strip

$$= 2.5 \text{ m} \left\{ \frac{(2.5 \text{ m}) \sin 60^\circ - x}{(2.5 \text{ m}) \sin 60^\circ} \right\}$$

$$= 2.5 \text{ m} - x \operatorname{cosec} 60^\circ$$

$\therefore$ Second moment of area about XX

$$= \int_0^{(2.5 \text{ m}) \sin 60^\circ} (2.5 \text{ m} - x \operatorname{cosec} 60^\circ) x^2 dx = \frac{2.5^4}{12} \sin^3 60^\circ \text{ m}^4$$

$$\text{First moment} = \text{Area} \times \frac{\text{Depth}}{3}$$

$$= \frac{1}{2} 2.5 \times 2.5 \sin 60^\circ \times \frac{2.5 \sin 60^\circ}{3} \text{ m}^3 = \frac{2.5^3}{6} \sin^2 60^\circ \text{ m}^3$$

$$\therefore \text{Depth of C.P.} = \frac{2.5}{2} \sin 60^\circ \text{ m} = \frac{\text{Depth}}{2}$$

$\therefore$ Thrust is equally divided between XX and bottom.

Thrust = Area $\times$ Pressure at centroid

$$= \frac{1}{2} 2.5^2 \sin 60^\circ \times 1000 \times 9.81 \times \frac{2.5 \sin 60^\circ}{3} \text{ N} = 19\,160 \text{ N}$$

$\therefore$ Load at bottom = 9580 N; at each upper corner 4790 N

2.12 Let shaft be at depth h below free surface. Then force on disc

$$= \pi R^2 \rho g h.$$

By parallel axes theorem, 2nd moment of area about free

$$\text{surface} = \pi R^4 / 4 + \pi R^2 h^2.$$

$$\text{1st moment of area about free surface} = \pi R^2 h$$

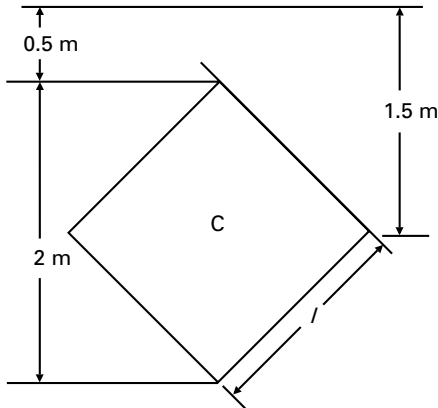
$$\therefore \text{Depth of C.P.} = \frac{R^2}{4h} + h \text{ below free surface, that is, } R^2/4h \text{ below shaft}$$

$\therefore$ Turning moment on shaft

$$= \pi R^2 \rho g h \times \frac{R^2}{4h} = \frac{\pi R^4 \rho g}{4} \quad [\text{independent of } h]$$

$$= \frac{\pi (0.6 \text{ m})^4 1000 \text{ kg} \cdot \text{m}^{-3} \times 9.81 \text{ N} \cdot \text{kg}^{-1}}{4} = 999 \text{ N} \cdot \text{m}$$

2.13



Force on plate

$$\begin{aligned}
 &= 1150 \text{ kg} \cdot \text{m}^{-3} \\
 &\quad \times 9.81 \text{ N} \cdot \text{kg}^{-1} \\
 &\quad \times 1.5 \text{ m} (\sqrt{2} \text{ m})^2 \\
 &= 33.84 \text{ kN}
 \end{aligned}$$

$$(Ak^2)_{c, \perp \text{ plate}} = \frac{Al^2}{6}$$

$$\therefore (Ak^2)_{c, \text{ diagonal}} = \frac{Al^2}{12}$$

since diagonals are perpendicular

 $\therefore$ Depth of C.P. below free surface

$$= \frac{(Al^2/12) + A\bar{y}^2}{A\bar{y}} = \bar{y} + \frac{l^2}{12\bar{y}} = \left\{ 1.5 + \frac{(\sqrt{2})^2}{12 \times 1.5} \right\} \text{ m}$$

= 1.611 m, that is, **1.111 m from top of aperture** $\therefore$ Total moment about hinge = $33.84 \text{ kN} \times 1.111/\sqrt{2} \text{ m}$

$$= 26.59 \text{ kN} \cdot \text{m}$$

2.14 Width of gates = $(3 \text{ m}) \sec 30^\circ = 3.464 \text{ m}$

Thrust on 'deep' side of gate

$$= (1000 \times 9.81 \times 4.5)(9 \times 3.464) \text{ N} = 1.376 \text{ MN}$$

Trust on 'shallow' side of gate

$$= (1000 \times 9.81 \times 1.35)(2.7 \times 3.464) \text{ N}$$

$$= \underline{0.124 \text{ MN}}$$

Net thrust = $(1.376 - 0.124) \text{ MN} = 1.252 \text{ MN}$

$$\therefore \text{ Force between gates} = \frac{1.252 \text{ MN}}{2 \sin 30^\circ} = 1.252 \text{ MN}$$

Resultant force F acts at height y given by

$$F_1 \frac{h_1}{3} - F_2 \frac{h_2}{3} = Fy, \text{ since } F_1, F_2 \text{ act at } \frac{2}{3}h_1, \frac{2}{3}h_2 \text{ below free surfaces}$$

$$\therefore y = \frac{1.376 \times 9/3 - 0.124 \times 2.7/3}{1.252} \text{ m} = 3.208 \text{ m}$$

Total hinge reaction R also acts at this height.