

Solution 1.8

We are given that $\sigma = y \text{ fcn}(M, I)$ and we are *not* to study up on strength of materials but only to use dimensional reasoning. For homogeneity, the right hand side must have dimensions of stress, that is,

$$\{\sigma\} = \{y\} \{\text{fcn}(M, I)\}, \quad \text{or:} \quad \left\{ \frac{M}{LT^2} \right\} = \{L\} \{\text{fcn}(M, I)\}$$

or: the function must have dimensions $\{\text{fcn}(M, I)\} = \left\{ \frac{M}{L^2 T^2} \right\}$

Therefore, to achieve dimensional homogeneity, we somehow must combine bending moment, whose dimensions are $\{ML^2 T^{-2}\}$, with area moment of inertia, $\{I\} = \{L^4\}$, and end up with $\{ML^{-2} T^{-2}\}$. Well, it is clear that $\{I\}$ contains neither mass $\{M\}$ nor time $\{T\}$ dimensions, but the bending moment contains both mass and time and in exactly the combination we need, $\{MT^{-2}\}$. Thus it must be that σ is proportional to M also. Now we have reduced the problem to:

$$\sigma = yM \text{ fcn}(I), \quad \text{or} \quad \left\{ \frac{M}{LT^2} \right\} = \{L\} \left\{ \frac{ML^2}{T^2} \right\} \{\text{fcn}(I)\}, \quad \text{or:} \quad \{\text{fcn}(I)\} = \{L^{-4}\}$$

We need just enough I 's to give dimensions of $\{L^{-4}\}$: we need the formula to be exactly *inverse* in I . The correct dimensionally homogeneous beam bending formula is thus:

$$\sigma = C \frac{My}{I}, \quad \text{where } \{C\} = \{\text{unity}\} \quad \text{Ans.}$$

The formula admits to an arbitrary dimensionless constant C whose value can only be obtained from known data. Convert stress into English units: $\sigma = (75 \text{ MPa})/(6894.8) = 10880 \text{ lbf/in}^2$. Substitute the given data into the proposed formula:

$$\sigma = 10880 \frac{\text{lbf}}{\text{in}^2} = C \frac{My}{I} = C \frac{(2900 \text{ lbf} \cdot \text{in})(1.5 \text{ in})}{0.4 \text{ in}^4}, \quad \text{or:} \quad C \approx \mathbf{1.00} \quad \text{Ans.}$$

The data show that $C = 1$, or $\sigma = My/I$, our old friend from strength of materials.