

Solution 1.61

For air at 20°C take $\mu \approx 1.8\text{E-}5 \text{ kg/m}\cdot\text{s}$. Let A be the bottom area of the puck, $A = \pi D^2/4$. Let x be in the direction of travel. Then the only force acting in the x direction is the air drag resisting the motion, assuming a linear velocity distribution in the air:

$$\sum F_x = -\tau A = -\mu \frac{V}{h} A = m \frac{dV}{dt}, \quad \text{where } h = \text{air film thickness}$$

Separate the variables and integrate to find the velocity of the decelerating puck:

$$\int_{V_0}^V \frac{dV}{V} = -K \int_0^t dt, \quad \text{or} \quad V = V_0 e^{-Kt}, \quad \text{where } K = \frac{\mu A}{mh}$$

Integrate again to find the displacement of the puck:

$$x = \int_0^t V dt = \frac{V_0}{K} [1 - e^{-Kt}]$$

Apply to the particular case given: air, $\mu \approx 1.8\text{E-}5 \text{ kg/m}\cdot\text{s}$, $m = 50 \text{ g}$, $D = 9 \text{ cm}$, $h = 0.12 \text{ mm}$, $V_0 = 10 \text{ m/s}$. First evaluate the time-constant K :

$$K = \frac{\mu A}{mh} = \frac{(1.8\text{E-}5 \text{ kg/m}\cdot\text{s})[(\pi/4)(0.09 \text{ m})^2]}{(0.050 \text{ kg})(0.00012 \text{ m})} \approx 0.0191 \text{ s}^{-1}$$

(a) When the puck slows down to 1 m/s, we obtain the time:

$$V = 1 \text{ m/s} = V_0 e^{-Kt} = (10 \text{ m/s}) e^{-(0.0191 \text{ s}^{-1})t}, \quad \text{or} \quad t \approx \mathbf{121 \text{ s}} \quad \text{Ans. (a)}$$

(b) The puck will stop completely only when $e^{-Kt} = 0$, or: $t = \infty$ Ans. (b)

(c) For part (a), the puck will have travelled, in 121 seconds,

$$x = \frac{V_0}{K} (1 - e^{-Kt}) = \frac{10 \text{ m/s}}{0.0191 \text{ s}^{-1}} [1 - e^{-(0.0191)(121)}] \approx \mathbf{472 \text{ m}} \quad \text{Ans. (c)}$$

This may perhaps be a little unrealistic. But the air-hockey puck *does* decelerate slowly!