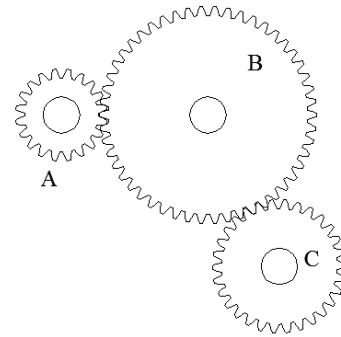


Problem 2.1

Given Information:

Gear A (80 mm diameter) has a constant rotational speed of 145 rpm clockwise.



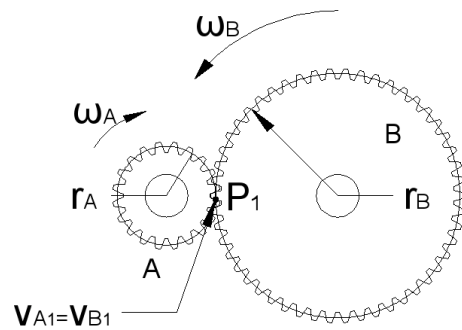
Required:

What is the rotational speed and direction of gears B (200 mm diameter) and C (120 mm diameter)?

Solution:

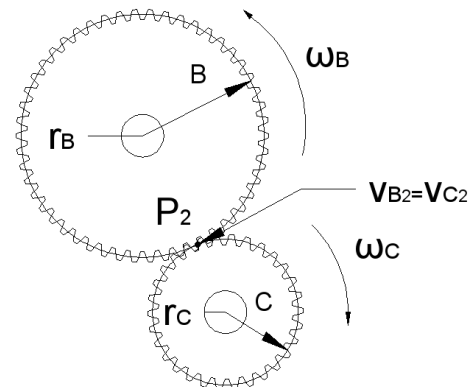
Consider the contact point 1 where gears A and B meet

$$\begin{aligned} V_{A_1} &= V_{B_1} \\ \omega_A r_A &= \omega_B r_B \\ \omega_B &= \omega_A \left(\frac{r_A}{r_B} \right) = \omega_A \left(\frac{D_A/2}{D_B/2} \right) = \omega_A \left(\frac{D_A}{D_B} \right) \\ \omega_B &= (145 \text{ rpm}) \left(\frac{80 \text{ mm}}{200 \text{ mm}} \right) = 58 \text{ rpm} \end{aligned}$$



Similarly, consider the contact point 2 where gears B and C meet

$$\begin{aligned} V_{B_2} &= V_{C_2} \\ \omega_B r_B &= \omega_C r_C \\ \omega_C &= \omega_B \left(\frac{r_B}{r_C} \right) = \omega_B \left(\frac{D_B}{D_C} \right) \\ \omega_C &= (58 \text{ rpm}) \left(\frac{200 \text{ mm}}{120 \text{ mm}} \right) = 96.67 \text{ rpm} \end{aligned}$$

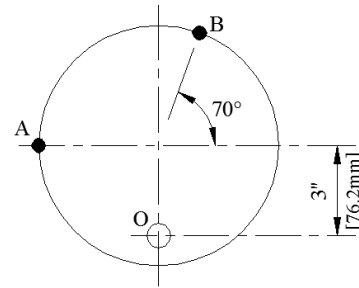


Answer: $\omega_B = 58 \text{ rpm ccw}$
 $\omega_C = 96.67 \text{ cw}$

Problem 2.2

Given Information:

The disk shown has a radius of 4 inches [101.6 mm] and rotates about the fixed point O at a constant rate of 75 rpm clockwise.



Required:

Determine the magnitude and direction of the velocities of points A and B.

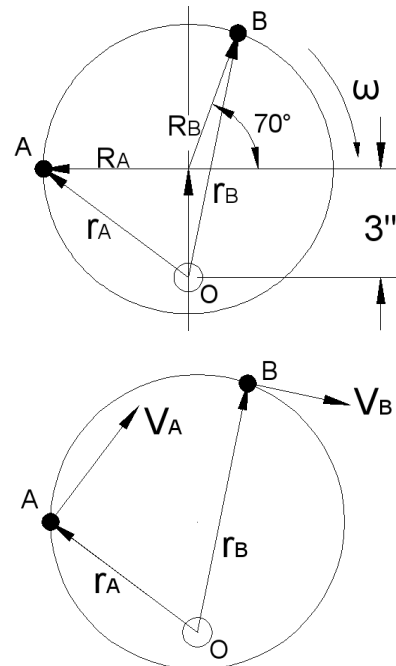
Solution:

The velocity of disk at point A is

$$\begin{aligned}
 \mathbf{V}_A &= \boldsymbol{\omega} \times \mathbf{r}_A \\
 \mathbf{V}_A &= \boldsymbol{\omega} \times (\mathbf{r}_O + \mathbf{R}_A) \\
 \mathbf{V}_A &= \left[\left(-75 \frac{\text{rev}}{\text{min}} \right) \hat{k} \left(\frac{\text{min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \right] \\
 &\quad \times [(0\hat{i} + r_o\hat{j}) + (-R\hat{i} + 0\hat{j})] \\
 \mathbf{V}_A &= \left[\left(-75 \frac{\text{rev}}{\text{min}} \right) \hat{k} \left(\frac{\text{min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \right] \\
 &\quad \times [(0\hat{i} + r_o\hat{j}) + (-R\hat{i} + 0\hat{j})] \\
 \mathbf{V}_A &= \left[\left(-\frac{75\pi}{30} \hat{k} \right) \frac{\text{rad}}{\text{s}} \right] \times [(-R\hat{i} + r_o\hat{j})] \\
 \mathbf{V}_A &= \left[\left(-\frac{75\pi}{30} \hat{k} \right) \frac{\text{rad}}{\text{s}} \right] \times [(-4\hat{i} + 3\hat{j})] \\
 \mathbf{V}_A &= (23.56\hat{i} + 31.42\hat{j}) \text{ in/s}
 \end{aligned}$$

The velocity of disk at point B is

$$\begin{aligned}
 \mathbf{V}_B &= \boldsymbol{\omega} \times \mathbf{r}_B \\
 \mathbf{V}_B &= \boldsymbol{\omega} \times (\mathbf{r}_O + \mathbf{R}_B) \\
 \mathbf{V}_B &= \left[\left(-\frac{75\pi}{30} \hat{k} \right) \frac{\text{rad}}{\text{s}} \right] \times [(r_o\hat{j}) + (R \cos 70^\circ \hat{i} + R \sin 70^\circ \hat{j})] \\
 \mathbf{V}_B &= \left[\left(-\frac{75\pi}{30} \hat{k} \right) \frac{\text{rad}}{\text{s}} \right] \times \{[(4 \cos 70^\circ)\hat{i} + (3 + 4 \sin 70^\circ)\hat{j}]\} \\
 \mathbf{V}_B &= (53.09\hat{i} - 10.76\hat{j}) \text{ in/s}
 \end{aligned}$$



$$V_A = 39.27 \frac{\text{in}}{\text{s}};$$

$$\theta_A = 53.16^\circ \nearrow$$

Answer:

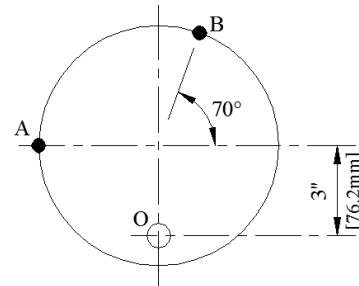
$$V_B = 54.17 \frac{\text{in}}{\text{s}};$$

$$\theta_B = -11.46^\circ \searrow$$

Problem 2.3

Given Information:

The disk shown has a radius of 4 inches [101.6 mm] and rotates about the fixed point O at a constant rate of 75 rpm clockwise.



Required:

Determine the magnitude and direction of the normal and tangential accelerations of point A.

Solution:

The normal acceleration of point A is $\mathbf{a}_{A,n} = \frac{V_A^2}{r_{O/A}} \mathbf{e}_n$, where $\mathbf{e}_n$ is the unit vector that indicates the position of point A w.r.t the fixed point O. This unit vector is equal to

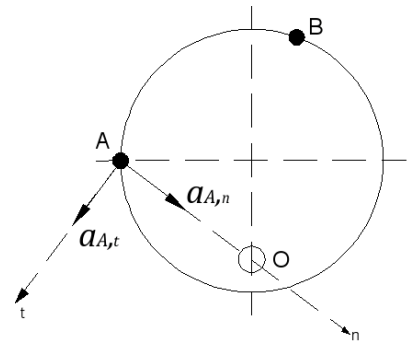
$$\mathbf{e}_n = \frac{\mathbf{r}_{O/A}}{r_{O/A}} = \frac{(4\hat{i} - 3\hat{j})\text{in}}{5\text{in}} = 0.8\hat{i} - 0.6\hat{j}$$

Moreover, from problem 2.2, $\mathbf{V}_A = (23.56\hat{i} + 31.42\hat{j}) \text{ in/s} \Rightarrow V_A = 39.27 \text{ in/s}$. Thus, the normal acceleration of point A is

$$\mathbf{a}_{A,n} = \frac{V_A^2}{r_{O/A}} \mathbf{e}_n = \frac{\left(\frac{39.27 \text{ in}}{\text{s}}\right)^2}{(5 \text{ in})} (0.8\hat{i} - 0.6\hat{j}) = (246.7\hat{i} - 185.06\hat{j}) \text{ in/s}^2$$

In addition, the tangential acceleration of point A is equal to zero because the disk rotates about point O with constant angular velocity. That is, the angular acceleration of the disk is equal to zero.

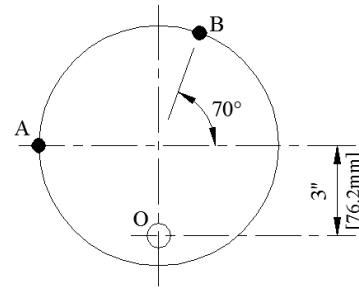
Answer: $a_{A,n} = 308.4 \text{ in/s}^2 @ \theta_A = 36.88^\circ \searrow$
 $a_{A,t} = 0$



Problem 2.4

Given Information:

At the instant shown, the disk shown has an angular velocity of 10 rad/s clockwise and is accelerating at a rate of 1.4 rad/s².



Required:

Determine the velocity and acceleration of point B.

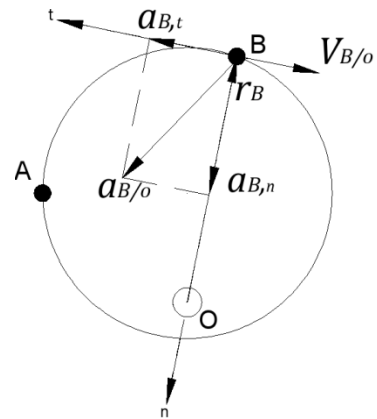
Solution:

The acceleration of point B w.r.t fixed point O is

$$\begin{aligned} \mathbf{a}_{B/O} &= \mathbf{a}_{B,t} + \mathbf{a}_{B,n} \\ \mathbf{a}_{B/O} &= (\alpha \times \mathbf{r}_B) + \left(\frac{V_B^2}{r_B} \right) \mathbf{e}_n \end{aligned}$$

where $\mathbf{e}_n$ is the unit vector that indicates the position of point B w.r.t the fixed point O, and is equal to

$$\begin{aligned} \mathbf{e}_n &= \frac{\mathbf{r}_B}{r_B} = \frac{[(-4 \cos 70^\circ)\hat{i} - (3 + 3 \sin 70^\circ)\hat{j}]\text{in}}{6.970 \text{ in}} \\ &= -0.199\hat{i} - 0.98\hat{j} \end{aligned}$$



Furthermore, the velocity of point B w.r.t the fixed point O is equal to

$$\begin{aligned} \mathbf{V}_{B/O} &= \boldsymbol{\omega} \times \mathbf{r}_B = \left[(-10\hat{k}) \frac{\text{rad}}{\text{s}} \right] \times [(4 \cos 70^\circ)\hat{i} + (3 + 3 \sin 70^\circ)\hat{j}]\text{in} \\ \mathbf{V}_{B/O} &= (67.7\hat{i} - 13.7\hat{j}) \text{ in/s} \Rightarrow V_{B/O} = 68.98 \text{ in/s} \end{aligned}$$

Substituting the speed and unit vector of point B w.r.t the fixed point O into the acceleration equation

$$\begin{aligned} \mathbf{a}_{B/O} &= \left[(-1.4\hat{k}) \frac{\text{rad}}{\text{s}^2} \right] \times [(1.37\hat{i} + 6.76\hat{j})\text{in}] + \left[\frac{(68.98 \text{ in/s})^2}{(6.90 \text{ in})} \times (-0.199\hat{i} - 0.98\hat{j}) \right] \\ \mathbf{a}_{B/O} &= \left[(9.464\hat{i} - 1.918\hat{j}) \frac{\text{in}}{\text{s}^2} \right] + [(-137.23\hat{i} - 675.8\hat{j}) \text{ in/s}^2] \\ \mathbf{a}_{B/O} &= (-127.77\hat{i} - 677.72\hat{j}) \text{ in/s}^2 \end{aligned}$$

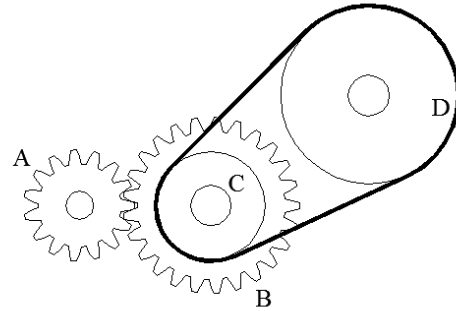
$$\text{Answer: } \mathbf{V}_{B/O} = 68.98 \text{ in/s @ } 11.46^\circ \searrow$$

$$\mathbf{a}_{B/O} = 689.7 \text{ in/s}^2 @ 1185.3^\circ$$

Problem 2.5

Given Information:

At the instant shown the 5 inch radius gear A rotates counterclockwise at a rate of 8 rad/s and is decelerating at a rate of 2.3 rad/s^2 . Gear A drives the 8.4 inch radius gear B. The 5.5 inch radius pulley C is on the same shaft as gear B. A belt around pulley C drives pulley D, which has a 9 inch radius.



Required:

What is the rotational velocity and acceleration of pulley D?

Solution:

Let point 1 be intersection of gears A and B. The speed of gear A at point 1 is equal to the speed of gear B at the same point

$$V_{A_1} = V_{B_1}$$

$$r_A \omega_A = r_B \omega_B$$

where r represents the radius of each pulley. Moreover, pulley C rotates with the same speed as gear B. Substituting this speed and the radii values yields

$$\omega_B = \left(\frac{r_A}{r_B}\right) \omega_A = \omega_C = \left(\frac{5 \text{ in}}{8.4 \text{ in}}\right) (8 \text{ rad/s})$$

$$\omega_C = 4.762 \text{ rad/s}$$

Furthermore, because a belt connects pulleys C and D, the pulleys have the same speed as the belt at all points of contact. Thus

$$V_C = V_D$$

$$r_C \omega_C = r_D \omega_D$$

$$\omega_D = \left(\frac{r_C}{r_D}\right) \omega_C = \left(\frac{5.5 \text{ in}}{9 \text{ in}}\right) (4.762 \text{ rad/s})$$

$$\omega_D = 2.91 \text{ rad/s}$$

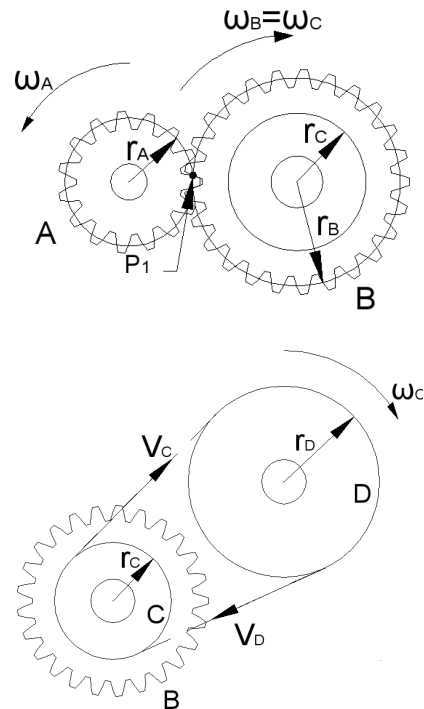
and by inspection, pulley D rotates in the clockwise direction. Similarly, the magnitudes of the tangential accelerations of gears A and B at contact point 1 are equal

$$a_{A_1,t} = a_{B_1,t}$$

$$r_A \alpha_A = r_B \alpha_B$$

$$\alpha_B = \alpha_A \left(\frac{r_A}{r_B}\right) = (-2.3 \text{ rad/s}^2) \left(\frac{5 \text{ in}}{8.4 \text{ in}}\right)$$

$$\alpha_B = -1.37 \text{ rad/s}^2$$



Also, because pulley C is attached to gear B, they both have the same angular acceleration. Similarly, pulleys C and D have the same angular acceleration because the belt connects them

$$\alpha_B = \alpha_C = \alpha_D = -1.37 \text{ rad/s}^2$$

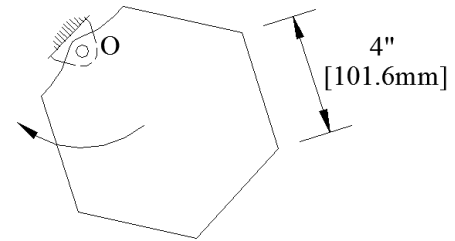
Note that the negative sign means pulley D is decelerating.

Answer: $\omega_D = 2.91 \text{ rad/s CW}$
 $\alpha_D = 1.37 \text{ rad/s}^2 \text{ CCW}$

Problem 2.6

Given Information:

The hexagon shown is rotating about point O at a speed of 5.6 rad/s clockwise and accelerating at a rate of 2.7 rad/s².

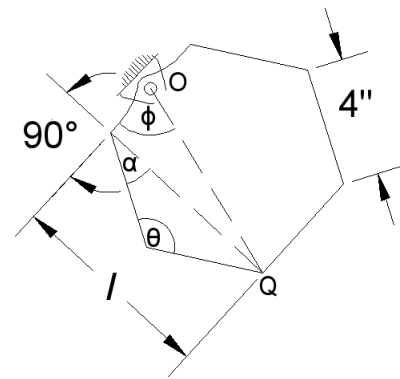


Required:

Determine the point along the perimeter that will have the highest velocity magnitude. Calculate the velocity and acceleration at that point.

Solution:

Let point Q be the point along the perimeter of the hexagon that has the highest speed. Because the speed of any point along the perimeter of the hexagon is directly proportional to its angular speed and the distance to that point from pivot point O, point Q will be located at the maximum distance from point O. That is, at either of the opposite vertices of the hexagon



$$\theta = \frac{(n-2)180^\circ}{n} = \frac{(6-2)180^\circ}{6} = 120^\circ$$

Applying the law of cosines to find the distance between opposite vertices, l

$$\frac{\sin \theta}{l} = \frac{\sin \alpha}{4 \text{ in}}$$

$$l = (4 \text{ in}) \left[\frac{\sin(120^\circ)}{\sin(120^\circ - 90^\circ)} \right] = 6.928 \text{ in}$$

To find the distance from point O to point Q

$$r_{Q/O} = \sqrt{l^2 + (2 \text{ in})^2} = \sqrt{(6.928 \text{ in})^2 + (2 \text{ in})^2} = 7.211 \text{ in}$$

To calculate the velocity of point Q, the position vector $\mathbf{r}_{Q/O}$ is required

$$\mathbf{r}_{Q/O} = r_{Q/O}(\cos \phi \hat{i} + \sin \phi \hat{j})$$

where $\phi = \tan^{-1}\left(\frac{l}{2 \text{ in}}\right) = 73.89^\circ$. As a result,

$$\mathbf{r}_{Q/O} = (7.211 \text{ in})[\cos(73.89^\circ) \hat{i} + \sin(73.89^\circ) \hat{j}] = (2\hat{i} + 6.93\hat{j}) \text{ in}$$

and the velocity of point Q is

$$\mathbf{V}_Q = \boldsymbol{\omega} \times \mathbf{r}_{Q/O}$$

$$\mathbf{V}_Q = \left[(-5.6 \hat{k}) \frac{\text{rad}}{\text{s}} \right] \times \{ (7.21 \text{ in}) [\cos(73.89^\circ) \hat{i} + \sin(73.89^\circ) \hat{j}] \}$$

$$\mathbf{V}_Q = (38.80 \hat{i} - 11.21 \hat{j}) \frac{\text{in}}{\text{s}}$$

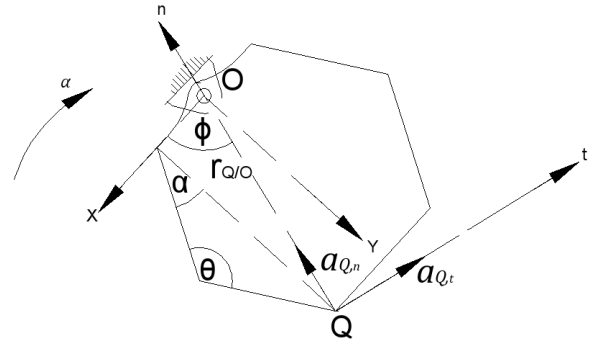
To find the acceleration of point Q

$$\mathbf{a}_Q = \mathbf{a}_{Q,n} + \mathbf{a}_{Q,t}$$

$$\mathbf{a}_Q = \left\{ \left(\frac{V_Q^2}{r_{Q/O}} \right) [-\cos(\phi) - \sin(\phi)] \right\} + (\boldsymbol{\alpha} \times \mathbf{r}_{Q/O})$$

$$\mathbf{a}_Q = \left\{ \frac{(40.4 \text{ in/s})^2}{(7.21 \text{ in})} [-\cos(73.89^\circ) - \sin(73.89^\circ)] \right\} + \left\{ \left[(-2.7 \hat{k}) \frac{\text{rad}}{\text{s}} \right] \times [(2 \hat{i} + 6.93 \hat{j}) \text{ in}] \right\}$$

$$\mathbf{a}_Q = (-44.21 \hat{i} - 222.9 \hat{j}) \frac{\text{in}}{\text{s}^2}$$

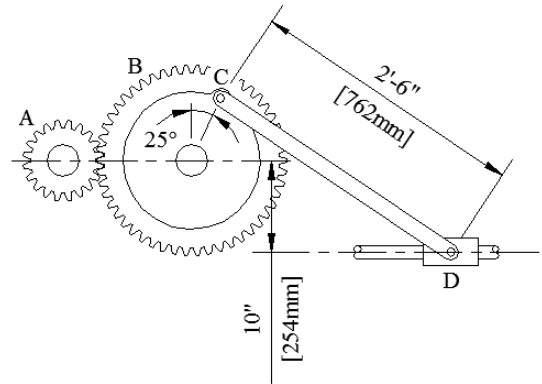


Answer: $V_Q = 40.4 \text{ in/s} @ 16.11^\circ \searrow$
 $a_Q = 227.2 \text{ in/s}^2 @ 78.7^\circ \swarrow$

Problem 2.7

Given Information:

In the mechanism shown the 8 inch diameter gear A turns at a constant speed of 1100 rpm clockwise and drives the 20 inch diameter gear B. Attached to gear B is a 15 inch diameter disk. A connecting rod CD is attached to the center of gear B at point C and to a slider D. The slider D moves vertically along a guide. The connecting rod CD is 2'6" long. The distance from the center of gear B to the slider D is 10".



Required:

Determine the angular velocity of the connecting rod CD and the velocity of slider D for the instant shown.

Solution:

First, determine the angular speed of gear B

$$V_{A_1} = V_{B_1}$$

$$\omega_A r_A = \omega_B r_B$$

$$\omega_B = \omega_A \left(\frac{r_A}{r_B} \right) = \omega_A \left(\frac{D_A}{D_B} \right) = (1100 \text{ rpm}) \left(\frac{8 \text{ in}}{20 \text{ in}} \right)$$

$$\omega_B = 440 \text{ rpm CCW}$$

Then, determine the velocity of point C

$$\mathbf{V}_C = \omega_C \times \mathbf{r}_C$$

$$\mathbf{V}_C = \left[\left(440 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \frac{\text{min}}{60 \text{ s}} \right) \hat{k} \right] \times (7.5 \text{ in}) [\cos(25^\circ) \hat{j} + \sin(25^\circ) \hat{i}]$$

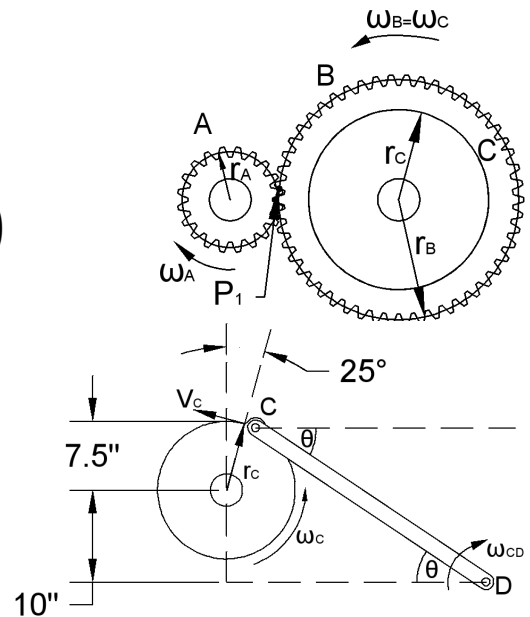
$$\mathbf{V}_C = (-313.2 \hat{i} + 146.1 \hat{j}) \text{ in/s}$$

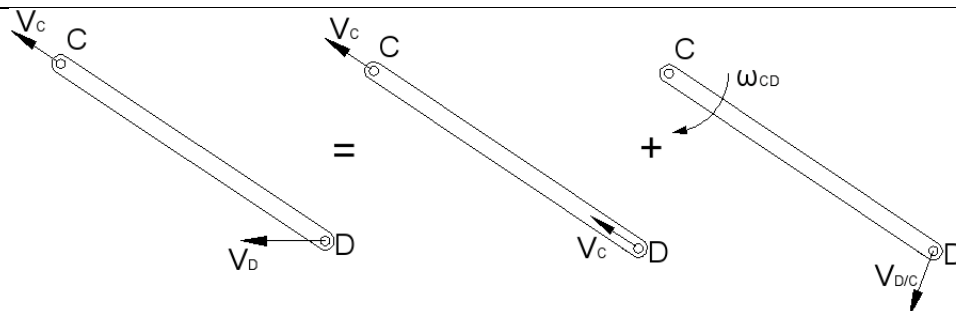
From trigonometry, θ can be found as follows

$$\sin \theta = \frac{[7.5 \cos(25^\circ) + 10] \text{ in}}{30 \text{ in}}$$

$$\theta = 34.05^\circ$$

Then, determine the velocity of the slider at point D:





$$\mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{D/C}$$

$$\mathbf{V}_D = \mathbf{V}_C + (\boldsymbol{\omega}_{CD} \times \mathbf{r}_{D/C}) = \mathbf{V}_C + [(-\omega_{CD})\hat{k}] \times [(r_{D/C})(\cos \theta \hat{i} - \sin \theta \hat{j})]$$

$$\mathbf{V}_D = [(-313.2\hat{i} + 146.1\hat{j}) \text{ in/s}] + \{[(-\omega_{CD})\hat{k}] \times (30 \text{ in})[\cos(34.05^\circ)\hat{i} - \sin(34.05^\circ)\hat{j}]\}$$

$$\mathbf{V}_D = (-313.2 - 16.8 \omega_{CD})\hat{i} + (146.1 - 24.86 \omega_{CD})\hat{j}$$

Because the slider moves in the horizontal direction, the vertical component of the velocity is zero

$$V_{Dy} = 0 = 146.1 - 24.86 \omega_{CD}$$

$$\omega_{CD} = 5.88 \text{ rad/s}$$

$$V_{Dx} = -313.2 - 16.8 \omega_{CD}$$

$$V_{Dx} = -313.2 - 16.8(5.88) = -412 \text{ in/s}$$

Answer: $\omega_{CD} = 5.88 \text{ rad/s CW}$
 $\mathbf{V}_D = 412 \text{ in/s @ } 180^\circ$

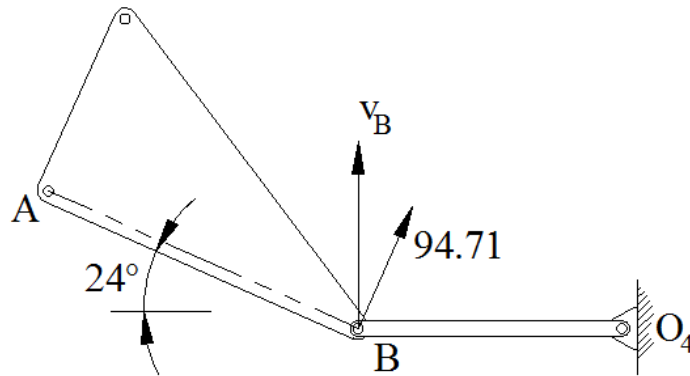
Problem 2.8

Given Information:

Continue Example Problem 2-4 to determine the rotation speed of link 4 [BO4].

Solution:

From problem 2-4



$$\cos 24 = \frac{94.71}{v_B} \Rightarrow v_B = \frac{94.71}{\cos 24} = 103.7 \frac{\text{in}}{\text{s}}$$

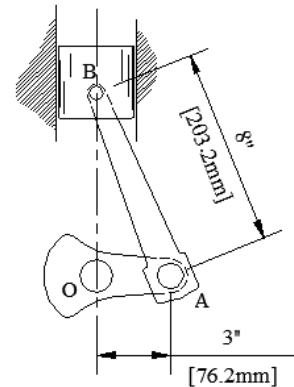
$$\omega_4 = \frac{v_B}{r_4} = \frac{103.7 \frac{\text{in}}{\text{s}}}{7 \text{ in}}$$

$$\text{Answer: } \omega_4 = 14.8 \frac{\text{rad}}{\text{s}} [CW]$$

Problem 2.9

Given Information:

The slider-crank mechanism shown in has a constant crank (link OA) rotation speed of 850 rpm clockwise.



Required:

Determine the velocity and acceleration of the piston (point B) at the instant shown.

Solution:

First, find the velocity of point A

$$\mathbf{V}_A = V_{Ax} + V_{Ay} = V_A(\cos \theta \hat{i} + \sin \theta \hat{j})$$

where $\theta = \cos^{-1}\left(\frac{3 \text{ in}}{8 \text{ in}}\right) = 68^\circ$, and

$$V_{Ay} = \omega_{OA} r_{OA} = V_A \sin \theta$$

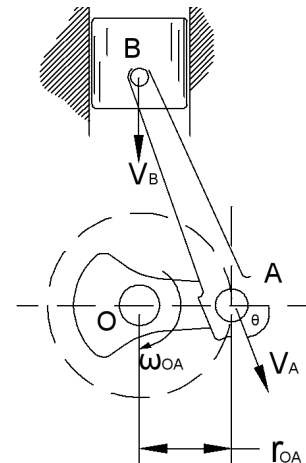
$$V_A \sin(68^\circ) = \left(850 \frac{\text{rev}}{\text{min}} \times \frac{\text{min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{\text{rev}}\right) (3 \text{ in})$$

$$V_A = 288.01 \text{ in/s}$$

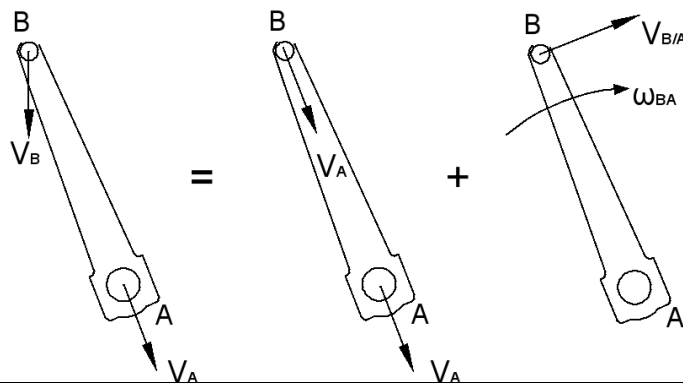
Substituting the speed of point A to obtain its velocity

$$\mathbf{V}_A = (288.01 \text{ in/s})[\cos(68^\circ) \hat{i} - \sin(68^\circ) \hat{j}]$$

$$\mathbf{V}_A = (107.9 \hat{i} - 267.04 \hat{j}) \text{ in/s}$$



Then, find the velocity of point B



$$\begin{aligned}
 \mathbf{V}_B &= \mathbf{V}_A + \mathbf{V}_{B/A} \\
 \mathbf{V}_B &= \mathbf{V}_A + (\boldsymbol{\omega}_{BA} \times \mathbf{r}_{B/A}) = \mathbf{V}_A + [(-\omega_{BA} \hat{k}) \times r_{B/A}(-\cos \theta \hat{i} + \sin \theta \hat{j})] \\
 \mathbf{V}_B &= [(107.9 \hat{i} - 267.04 \hat{j}) \text{ in/s}] + \{ [(-\omega_{BA} \hat{k}) \times (8 \text{ in})[-\cos(68^\circ) \hat{i} + \sin(68^\circ) \hat{j}]] \} \\
 \mathbf{V}_B &= (107.9 + 7.42 \omega_{BA}) \hat{i} + (-267.04 + 3 \omega_{BA}) \hat{j}
 \end{aligned}$$

Because the piston is constrained to move in the vertical direction only

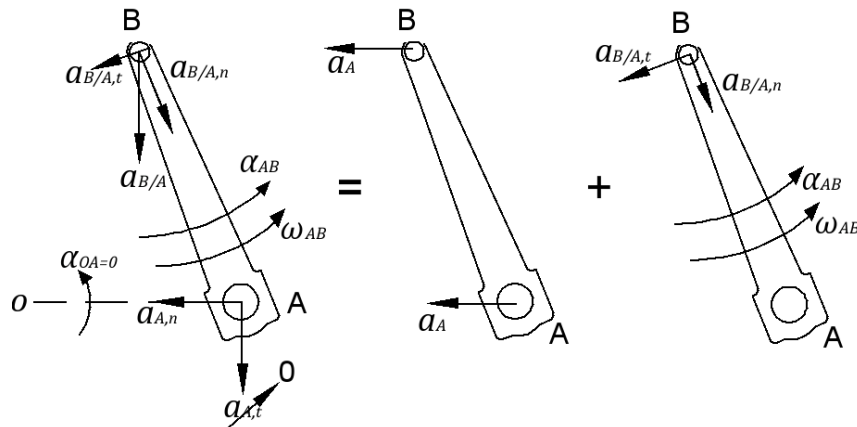
$$\begin{aligned}
 V_{B_x} &= 0 = 107.9 + 7.42 \omega_{BA} \\
 \omega_{BA} &= -14.54 \text{ rad/s}
 \end{aligned}$$

Substituting the angular speed into the velocity of the piston equation

$$\mathbf{V}_B = [-267.04 + 3(-14.54)] \hat{j} = (-310.7 \hat{j}) \text{ in/s}$$

Then, find the acceleration of the piston using the relative accelerations method

$$\begin{aligned}
 \mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_{A,n} + \mathbf{a}_{A,t} + \mathbf{a}_{B/A,n} + \mathbf{a}_{B/A,t} \\
 \mathbf{a}_B &= \boldsymbol{\omega}_{AO} \times (\boldsymbol{\omega}_{AO} \times \mathbf{r}_{A/O}) + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) + (\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A})
 \end{aligned}$$



$$\begin{aligned}
 \mathbf{a}_B &= [(-89.01 \hat{k}) \text{ rad/s}] \times \{ [(-89.01 \hat{k}) \text{ rad/s}] \times [(3 \hat{i}) \text{ in}] \} + [(14.54 \hat{k}) \text{ rad/s}] \\
 &\quad \times \{ [(14.54 \hat{k}) \text{ rad/s}] \times (8 \text{ in})[-\cos(68^\circ) \hat{i} + \sin(68^\circ) \hat{j}] \} + (\alpha_{AB} \hat{k}) \\
 &\quad \times \{ (\alpha_{AB} \hat{k}) \times (8 \text{ in})[-\cos(68^\circ) \hat{i} + \sin(68^\circ) \hat{j}] \} \\
 \mathbf{a}_B &= (-23134.8 - 7.42 \alpha_{AB}) \hat{i} + (-1569 - 3 \alpha_{AB}) \hat{j}
 \end{aligned}$$

Because the piston is constrained to move in the vertical direction only, the acceleration in the horizontal direction is zero

$$\begin{aligned}
 a_{B_x} &= 0 = -23134.8 - 7.42 \alpha_{AB} \\
 \alpha_{AB} &= -3117.9 \text{ rad/s}^2
 \end{aligned}$$

As a result, the acceleration is

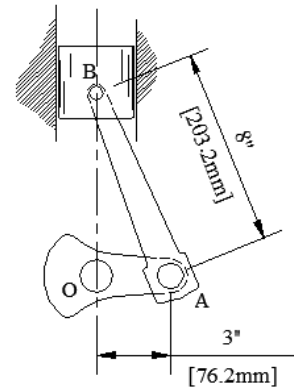
$$\begin{aligned}
 \mathbf{a}_B &= [-1569 - 3(-3117.9)] \hat{j} \\
 \mathbf{a}_B &= (7784.7 \hat{j}) \text{ in/s}^2
 \end{aligned}$$

Answer: $\mathbf{V}_B = 310 \text{ in/s @ } 270^\circ$
 $\mathbf{a}_B = 7784.7 \text{ in/s}^2 \text{ @ } 90^\circ$

Problem 2.10

Given Information:

The slider-crank mechanism is decelerating. At the instant shown the crank is rotating counterclockwise at a speed of 640 rpm. It is decelerating at a constant rate and comes to a complete stop in 1.2 minutes.



Required:

Determine the velocity and acceleration of the piston at the instant shown.

Solution:

First, find the velocity of point A at the given instant

$$\mathbf{V}_A = V_{Ax} + V_{Ay} = V_A(-\cos \theta \hat{i} + \sin \theta \hat{j})$$

where $\theta = \cos^{-1}\left(\frac{3 \text{ in}}{8 \text{ in}}\right) = 68^\circ$, and

$$V_{Ay} = \omega_{OA} r_{OA} = V_A \sin \theta$$

$$V_A \sin(68^\circ) = \left(640 \frac{\text{rev}}{\text{min}} \times \frac{\text{min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{\text{rev}}\right) (3 \text{ in})$$

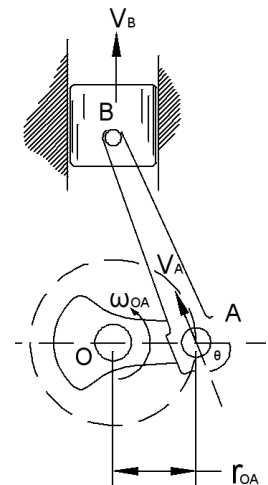
$$V_A = 216.9 \text{ in/s}$$

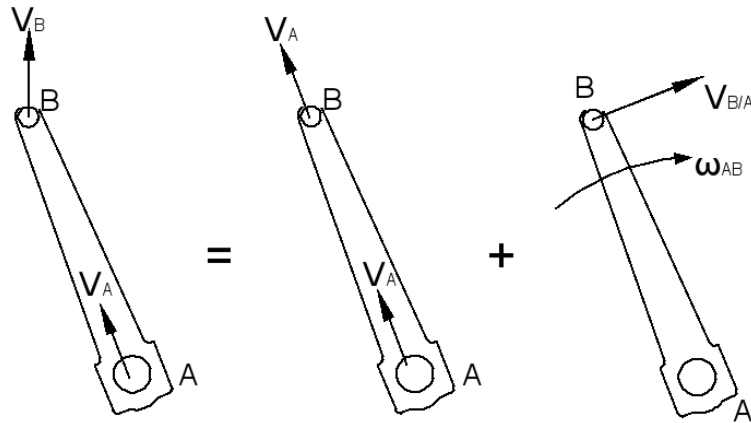
Substituting the speed of point A to obtain its velocity

$$\mathbf{V}_A = (216.9 \text{ in/s})[-\cos(68^\circ) \hat{i} + \sin(68^\circ) \hat{j}]$$

$$\mathbf{V}_A = (-81.23 \hat{i} + 201.1 \hat{j}) \text{ in/s}$$

Then, find the velocity of point B





$$\begin{aligned} \mathbf{V}_B &= \mathbf{V}_A + \mathbf{V}_{B/A} \\ \mathbf{V}_B &= \mathbf{V}_A + (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) = \mathbf{V}_A + [(-\omega_{BA} \hat{k}) \times r_{B/A}(-\cos \theta \hat{i} + \sin \theta \hat{j})] \\ \mathbf{V}_B &= [(-81.23 \hat{i} + 201.1 \hat{j}) \text{ in/s}] + \{[(-\omega_{BA} \hat{k}) \times (8 \text{ in})[-\cos(68^\circ) \hat{i} + \sin(68^\circ) \hat{j}]]\} \\ \mathbf{V}_B &= (-81.23 - 7.42 \omega_{AB}) \hat{i} + (201.1 - 3 \omega_{AB}) \hat{j} \end{aligned}$$

Because the piston is constrained to move in the vertical direction only

$$\begin{aligned} V_{B_x} = 0 &= -81.23 - 7.42 \omega_{AB} \\ \omega_{BA} &= -10.95 \text{ rad/s} \end{aligned}$$

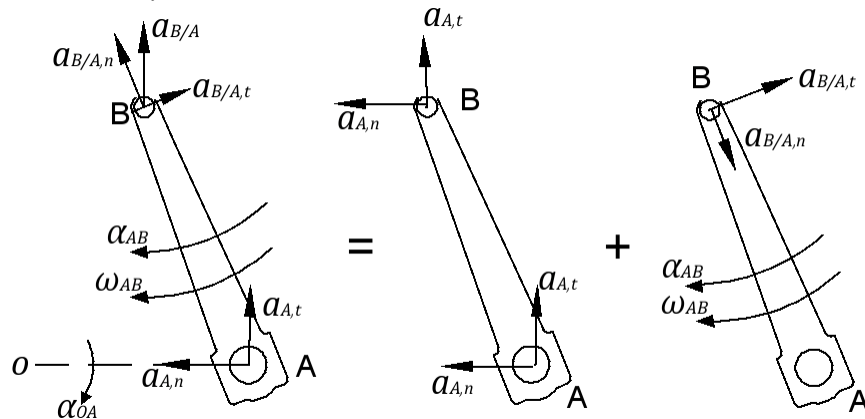
Substituting the angular speed into the velocity of the piston equation

$$\mathbf{V}_B = [201.1 - 3(-10.95)] \hat{j} = (234 \hat{j}) \text{ in/s}$$

The angular acceleration of arm OA is required to find the acceleration of the piston. Assuming that the mechanism is decelerating uniformly, the angular acceleration of arm OA can be calculated as

$$\alpha_{OA} = \frac{(\omega_f - \omega_o)_{OA}}{\Delta t} = \frac{(0 - 640 \frac{\text{rev}}{\text{min}})_{OA}}{(1.2 \text{ min})} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \left(\frac{\text{min}}{60 \text{ s}}\right)^2 = -0.931 \text{ rad/s}^2$$

And the acceleration of the piston is



$$\begin{aligned}
\mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_{A,n} + \mathbf{a}_{A,t} + \mathbf{a}_{B/A,n} + \mathbf{a}_{B/A,t} \\
\mathbf{a}_B &= \boldsymbol{\omega}_{AO} \times (\boldsymbol{\omega}_{OA} \times \mathbf{r}_{A/O}) + (\boldsymbol{\alpha}_{AO} \times \mathbf{r}_{A/O}) + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) + (\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A}) \\
\mathbf{a}_B &= [(-0.931\hat{k})\text{rad/s}^2] \times \{[(-0.931\hat{k})\text{rad/s}^2] \times [(3\hat{i})\text{in}]\} + [(67.02\hat{k})\text{rad/s}] \\
&\quad \times \{[(67.02\hat{k})\text{rad/s}] \times [(3\hat{i})\text{in}]\} + [(-10.95\hat{k})\text{rad/s}] \\
&\quad \times \{[(-10.95\hat{k})\text{rad/s}] \times (8\text{in})[-\cos(68^\circ)\hat{i} + \sin(68^\circ)\hat{j}]\} + (\alpha_{AB}\hat{k}) \\
&\quad \times \{(\alpha_{AB}\hat{k}) \times (8\text{in})[-\cos(68^\circ)\hat{i} + \sin(68^\circ)\hat{j}]\} \\
\mathbf{a}_B &= (-13115.4 - 7.42\alpha_{AB})\hat{i} + (-892.2 - 3\alpha_{AB})\hat{j}
\end{aligned}$$

Because the piston is constrained to move in the vertical direction only, the acceleration in the horizontal direction is zero

$$\begin{aligned}
a_{B_x} &= 0 = -13115.4 - 7.42\alpha_{AB} \\
\alpha_{AB} &= -1767.6\text{ rad/s}^2
\end{aligned}$$

As a result, the acceleration is

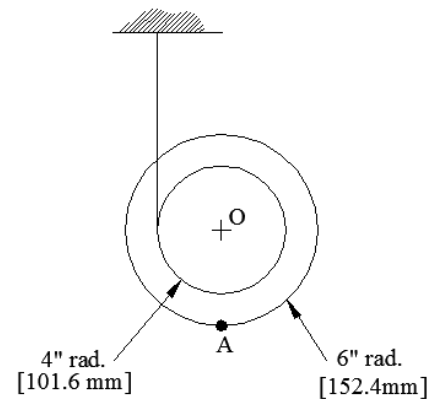
$$\begin{aligned}
\mathbf{a}_B &= [-892.2 - 3(-1767.6)]\hat{j} \\
\mathbf{a}_B &= (4411\hat{j})\text{in/s}^2
\end{aligned}$$

$$\begin{aligned}
\text{Answer: } \mathbf{V}_B &= 234\text{ in/s @ } 90^\circ \\
\mathbf{a}_B &= 4411\text{ in/s}^2 @ 90^\circ
\end{aligned}$$

Problem 2.11

Given Information:

The spool shown is unwinding such that point O moves straight down. At the instant shown the spool has an angular velocity of 2 rad/s and an angular acceleration of 3.5 rad/s² (both counterclockwise).



Required:

Find the acceleration of point O and point A.

Solution:

First, find the acceleration of the center of the spool using the relative accelerations method

$$\mathbf{a}_O = \mathbf{a}_P + \mathbf{a}_{O/P} = \mathbf{a}_P + \mathbf{a}_{O/P,n} + \mathbf{a}_{O/P,t}$$

$$\mathbf{a}_O = [(\omega_{A/O}^2 r_{O/P})\hat{i}] + \boldsymbol{\omega}_{A/O} \times (\boldsymbol{\omega}_{A/O} \times \mathbf{r}_{O/P}) + \boldsymbol{\alpha}_{A/O} \times \mathbf{r}_{O/P}$$

$$\mathbf{a}_O = \{[(2 \text{ rad/s})^2 (2 \text{ in})]\hat{i}\} + [(2\hat{k})\text{rad/s}] \times \{[(2\hat{k})\text{rad/s}] \times [(2\hat{i})\text{in}]\}$$

$$+ \{[(3.5\hat{k})\text{rad/s}^2] \times [(2\hat{i})\text{in}]\}$$

$$\mathbf{a}_O = (7\hat{j})\text{in/s}^2$$

Then, find the acceleration of point A noting that the center of the spool moves vertically only

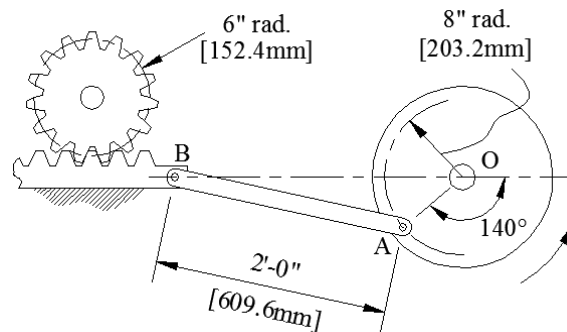
$$\begin{aligned}
 \mathbf{a}_A &= \mathbf{a}_O + \mathbf{a}_{A/O} = \mathbf{a}_O + \mathbf{a}_{A/O,n} + \mathbf{a}_{A/O,t} \\
 \mathbf{a}_A &= \mathbf{a}_O + \boldsymbol{\omega}_{A/O} \times (\boldsymbol{\omega}_{A/O} \times \mathbf{r}_{A/O}) + \boldsymbol{\alpha}_{A/O} \times \mathbf{r}_{A/O} \\
 \mathbf{a}_A &= [(7\hat{j})in/s^2] + [(2\hat{k})rad/s] \times \{[(2\hat{k})rad/s] \times [(-3\hat{j})in]\} \\
 &\quad + \{[(3.5\hat{k})rad/s^2] \times [(-3\hat{j})in]\} \\
 \mathbf{a}_A &= (10.5\hat{i} + 19\hat{j})in/s^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Answer:} \quad \mathbf{a}_O &= 7 \text{ in/s}^2 @ 90^\circ \\
 \mathbf{a}_A &= 21.71 \text{ in/s}^2 @ 61.1^\circ \nearrow
 \end{aligned}$$

Problem 2.12

Given Information:

The disk shown rotates about point O at a constant speed of 45 rpm. Connecting rod AB is attached to a rack that turns the 6 inch radius gear.



Required:

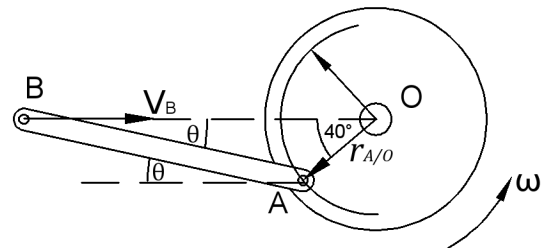
What is the rotational speed and rotational acceleration of the gear at the instant shown?

Solution:

First, find the angle θ using the law of sines

$$\frac{\sin(40^\circ)}{24 \text{ in}} = \frac{\sin(\theta)}{8 \text{ in}}$$

$$\theta = 12.37^\circ$$



Next, find the velocity of the linkage point B

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{B/A} = \omega \times \mathbf{r}_{A/O} + \omega_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{V}_B = \left[\left(45 \times \frac{2\pi}{60} \hat{k} \right) \text{ rad/s} \right] \times \{ (8 \text{ in}) [-\cos(40^\circ) \hat{i} - \sin(40^\circ) \hat{j}] \} + (\omega_{AB} \hat{k})$$

$$\times \{ (24 \text{ in}) [-\cos(12.37^\circ) \hat{i} + \sin(12.37^\circ) \hat{j}] \}$$

$$\mathbf{V}_B = (24.23 - 5.14\omega_{AB}) \hat{i} + (-28.9 - 23.44\omega_{AB}) \hat{j}$$

Because the rack is constrained to move in the horizontal direction only

$$V_{By} = 0 = -28.9 - 23.44\omega_{AB}$$

$$\omega_{AB} = -1.23 \text{ rad/s}$$

and

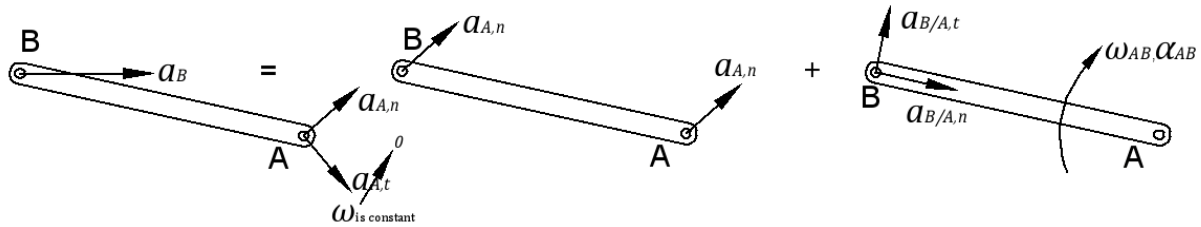
$$\mathbf{V}_B = [24.23 - 5.14(-1.23)] \hat{i}$$

$$\mathbf{V}_B = (30.57 \hat{i}) \text{ in/s}$$

Thus, the rotational speed of the gear at the instant shown is

$$\omega_G = \frac{V_B}{r_G} = \frac{30.57 \text{ in/s}}{6 \text{ in}} = 5.095 \text{ rad/s}$$

Similarly, to find the angular acceleration of the gear, determine the acceleration of linkage point B



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_{A,n} + \mathbf{a}_{B/A,n} + \mathbf{a}_{B/A,t}$$

$$\mathbf{a}_B = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/O}) + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) + (\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A})$$

$$\begin{aligned} \mathbf{a}_B &= [(4.71\hat{k})\text{rad/s}] \times \{[(4.71\hat{k})\text{rad/s}] \times \{(8\text{in})[-\cos(40^\circ)\hat{i} - \sin(40^\circ)\hat{j}]\}\} + [(-1.23\hat{k})\text{rad/s}] \\ &\quad \times \{[(-1.23\hat{k})\text{rad/s}] \times \{(24\text{in})[-\cos(12.37^\circ)\hat{i} + \sin(12.37^\circ)\hat{j}]\}\} + (\alpha_{AB}\hat{k}) \\ &\quad \times \{(24\text{in})[-\cos(12.37^\circ)\hat{i} + \sin(12.37^\circ)\hat{j}]\} \\ \mathbf{a}_B &= (171.84 - 5.14\alpha_{AB})\hat{i} + (106.38 - 23.44\alpha_{AB})\hat{j} \end{aligned}$$

Because the rack is constrained to move in the horizontal direction only

$$a_{B_y} = 0 = 106.38 - 23.44\alpha_{AB}$$

$$\alpha_{AB} = 4.54 \text{ rad/s}^2$$

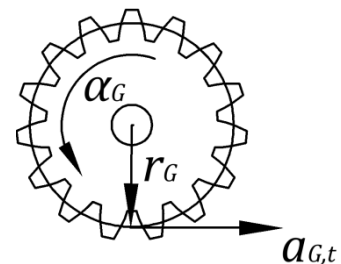
and

$$\mathbf{a}_B = [171.84 - 5.14(4.54)]\hat{i}$$

$$\mathbf{a}_B = (148.5\hat{i})\text{rad/s}^2$$

Finally, determine the angular acceleration of the gear noticing that

$$\begin{aligned} \mathbf{a}_{G,t} &= \boldsymbol{\alpha}_G \times \mathbf{r}_G = \mathbf{a}_B \\ \alpha_G &= \frac{a_B}{r_G} = \frac{148.5 \text{ in/s}^2}{6 \text{ in}} = 24.75 \text{ rad/s}^2 \end{aligned}$$

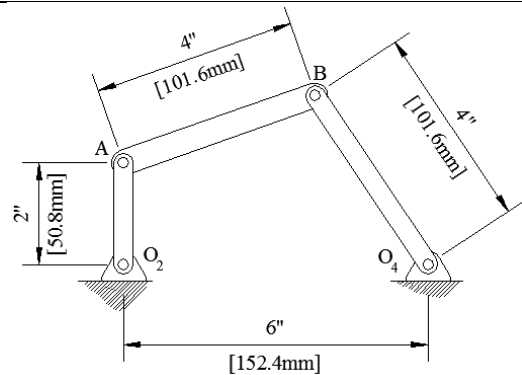


Answer: $\omega_G = 5.095 \text{ rad/s}$
 $\alpha_G = 24.75 \text{ rad/s}^2$

Problem 2.13

Given Information:

The linkage mechanism shown is driven by link O_2A , which rotates at a constant speed of 90 rpm clockwise.



Required:

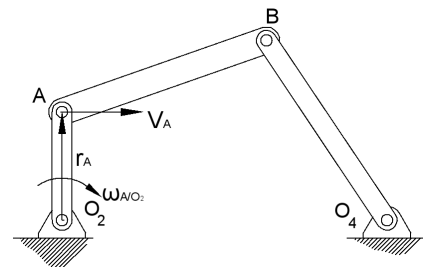
Determine the rotational velocity and acceleration of link O_4B in the position shown when the drive link is vertical.

Solution:

First, find the velocity of point A

$$\mathbf{V}_A = \left[\left(90 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \frac{\text{min}}{60 \text{ s}} \right) \hat{k} \right] \times [(2\hat{j})\text{in}]$$

$$\mathbf{V}_A = (18.85 \hat{i})\text{in/s}$$



Then, find the internal angles of the mechanism at the given instant

$$l = \sqrt{(6 \text{ in})^2 + (2 \text{ in})^2} = 2\sqrt{10} \text{ in}$$

Using the law of sines:

$$\frac{\sin \theta}{4 \text{ in}} = \frac{\sin(180^\circ - 2\theta)}{2\sqrt{10} \text{ in}}$$

$$\theta = 37.8^\circ$$

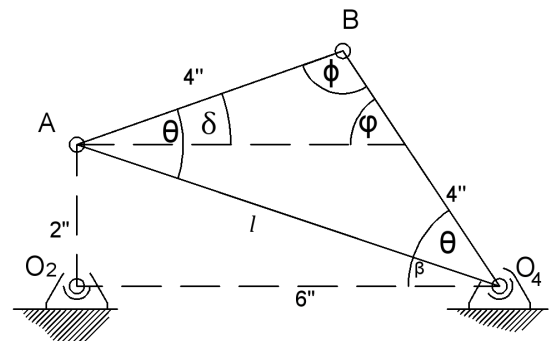
and

$$\phi = 180^\circ - 2\theta = 104.5^\circ$$

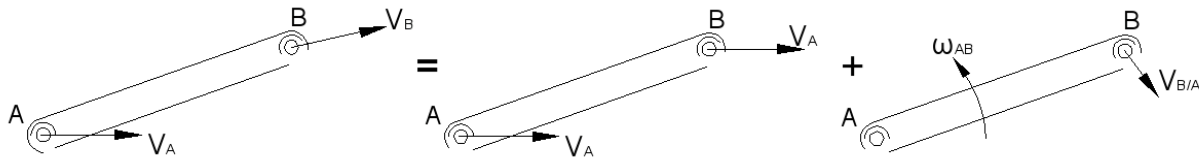
$$\beta = \tan^{-1} \left(\frac{2 \text{ in}}{6 \text{ in}} \right) = 18.43^\circ$$

$$\varphi = 180^\circ - \phi - (\theta - \beta) = 56.13^\circ$$

$$\delta = \theta - \beta = 19.37^\circ$$



Then, find the velocity of linkage AB at point B



$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{B/A} = \mathbf{V}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{V}_B = [(18.85 \hat{i}) \text{ in/s}] + \{(\omega_{AB} \hat{k}) \times (4 \text{ in})[\cos(19.37^\circ) \hat{i} + \sin(19.37^\circ) \hat{j}]\}$$

$$\mathbf{V}_B = (18.85 - 1.33\omega_{AB}) \hat{i} + (3.8\omega_{AB}) \hat{j} \quad \text{Eq. (1)}$$

Moreover, the velocity of point B can be found from linkage BO_4

$$\mathbf{V}_B = \boldsymbol{\omega}_{B/O_4} \times \mathbf{r}_{B/O_4}$$

$$\mathbf{V}_B = (\omega_{B/O_4} \hat{k}) \times \{(4 \text{ in}) \times [-\cos(56.13^\circ) \hat{i} + \sin(56.13^\circ) \hat{j}]\}$$

$$\mathbf{V}_B = (-3.32\omega_{B/O_4}) \hat{i} + (-2.23\omega_{B/O_4}) \hat{j} \quad \text{Eq. (2)}$$

Setting Eq. (1) equal to Eq. (2) in their respective directions

$$(18.85 - 1.33\omega_{AB}) \hat{i} = (-3.32\omega_{B/O_4}) \hat{i}$$

$$-1.33\omega_{AB} + 3.32\omega_{B/O_4} = -18.85$$

and

$$(3.8\omega_{AB}) \hat{j} = (-2.23\omega_{B/O_4}) \hat{j}$$

$$3.8\omega_{AB} + 2.23\omega_{B/O_4} = 0$$

Solving for the system of equations yields

$$\omega_{B/O_4} = -4.6 \text{ rad/s}$$

$$\omega_{AB} = 2.71 \text{ rad/s}$$

To find the angular acceleration of linkage BO_4 , first find the acceleration of point B

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_{A,n} + \mathbf{a}_{B/A,n} + \mathbf{a}_{B/A,t}$$

$$\mathbf{a}_B = (\boldsymbol{\omega}_{A/O_2} \times \mathbf{V}_A) + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) + (\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A})$$

$$\mathbf{a}_B = [(-9.425 \hat{k}) \text{ rad/s}] \times [(18.85 \hat{i}) \text{ in/s}] + [(2.706 \hat{k}) \text{ rad/s}] \times [(2.706 \hat{k}) \text{ rad/s}] \times \{(4 \text{ in})[\cos(19.37^\circ) \hat{i} + \sin(19.37^\circ) \hat{j}]\} + (\alpha_{AB} \hat{k}) \times \{(4 \text{ in})[\cos(19.37^\circ) \hat{i} + \sin(19.37^\circ) \hat{j}]\}$$

$$\mathbf{a}_B = (-27.63 - 1.33\alpha_{AB}) \hat{i} + (-187.41 + 3.77\alpha_{AB}) \hat{j} \quad \text{Eq. (3)}$$

Similarly, find an equation for the acceleration of point B from linkage BO_4

$$\begin{aligned}
 \mathbf{a}_B &= \mathbf{a}_{B/O_4,n} + \mathbf{a}_{B/O_4,t} \\
 \mathbf{a}_B &= \boldsymbol{\omega}_{B/O_4} \times (\boldsymbol{\omega}_{B/O_4} \times \mathbf{r}_{B/O_4}) + (\boldsymbol{\alpha}_{AB/O_4} \times \mathbf{r}_{B/O_4}) \\
 \mathbf{a}_B &= [(-4.6)rad/s] \times \{ [(-4.6\hat{k})rad/s] \times \{(4in)[- \cos(56.13^\circ)\hat{i} + \sin(56.13^\circ)\hat{j}] \} \} + (\alpha_{B/O_4}\hat{k}) \\
 &\quad \times \{(4in)[- \cos(56.13^\circ)\hat{i} + \sin(56.13^\circ)\hat{j}] \} \\
 \mathbf{a}_B &= (47.2 - 3.32\alpha_{B/O_4})\hat{i} + (-70.28 - 2.23\alpha_{B/O_4})\hat{j} \quad \mathbf{Eq. (4)}
 \end{aligned}$$

Setting Eq. (3) equal to Eq. (4) in their respective directions

$$\begin{aligned}
 (-27.63 - 1.33\alpha_{AB})\hat{i} &= (47.2 - 3.32\alpha_{B/O_4})\hat{i} \\
 -1.33\alpha_{AB} + 3.32\alpha_{B/O_4} &= 74.83
 \end{aligned}$$

and

$$\begin{aligned}
 (-187.41 + 3.77\alpha_{AB})\hat{j} &= (-70.28 - 2.23\alpha_{B/O_4})\hat{j} \\
 3.77\alpha_{AB} + 2.23\alpha_{B/O_4} &= 117.13
 \end{aligned}$$

Solving for the system of equations yields

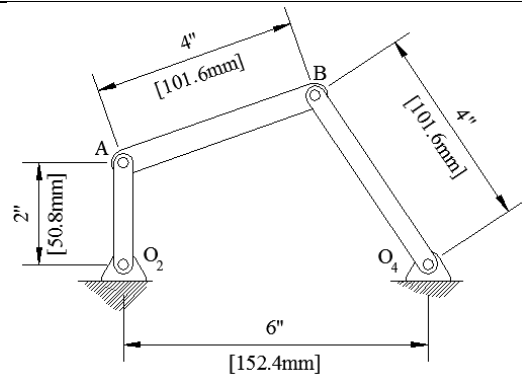
$$\alpha_{B/O_4} = 22.25 \text{ rad/s}^2$$

$$\begin{aligned}
 \text{Answer:} \quad \boldsymbol{\omega}_{B/O_4} &= 4.6 \text{ rad/s } CW \\
 \boldsymbol{\alpha}_{B/O_4} &= 22.25 \text{ rad/s}^2 \text{ CCW}
 \end{aligned}$$

Problem 2.14

Given Information:

The linkage mechanism shown in Figure 2.42 is driven by link O_2A , which rotates at a constant speed of 90 rpm clockwise.



Required:

Determine the velocity magnitude of the midpoint of link AB.

Solution:

First, find the velocity of the midpoint of link AB

$$\mathbf{V}_M = \mathbf{V}_A + \mathbf{V}_{M/A} = \mathbf{V}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{M/A}$$

where $\mathbf{V}_A$ and $\boldsymbol{\omega}_{AB}$ were found in problem 2.13. As a result

$$\begin{aligned} \mathbf{V}_B &= \mathbf{V}_A + \mathbf{V}_{B/A} = \mathbf{V}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} \\ \mathbf{V}_B &= [(18.85 \hat{i}) \text{ in/s}] + \{[(2.71 \hat{k}) \text{ rad/s}] \times (2 \text{ in})[\cos(19.37^\circ) \hat{i} + \sin(19.37^\circ) \hat{j}]\} \\ \mathbf{V}_B &= (17.05 \hat{i} + 5.125 \hat{j}) \text{ in/s} \end{aligned}$$

and

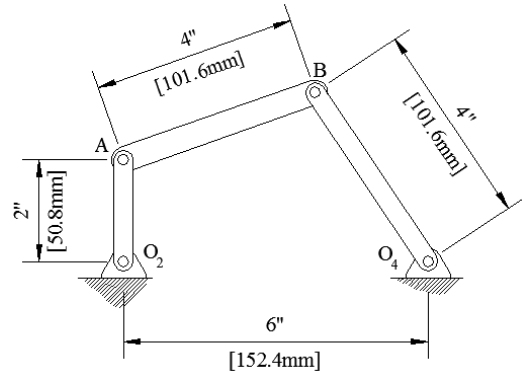
$$V_B = \sqrt{(17.05)^2 + (5.125)^2} \text{ in/s} = 17.80 \text{ in/s}$$

Answer: $V_B = 17.80 \text{ in/s}$

Problem 2.15

Given Information:

For the linkage shown the drive link O_2A rotates at 5 rad/s clockwise and is accelerating at a rate of 1.2 rad/s^2 .



Required:

Determine the rotational velocity and acceleration of link O_4B in the position shown.

Solution:

First, find the velocity and acceleration of point A

$$\mathbf{V}_A = \omega_{A/O_2} \times \mathbf{r}_A = [(5 \hat{k}) \text{ rad/s}] \times [(2 \hat{j}) \text{ in}]$$

$$\mathbf{V}_A = (10 \hat{i}) \text{ in/s}$$

$$\mathbf{a}_A = \mathbf{a}_{A,t} + \mathbf{a}_{A,n} = (\alpha_{A/O_2} \times \mathbf{r}_A) + (\omega_{A/O_2} \times \mathbf{V}_A)$$

$$\mathbf{a}_A = [(-1.2 \hat{k}) \text{ rad/s}^2] \times [(2 \hat{j}) \text{ in}] + [(-5 \hat{k}) \text{ rad/s}] \times [(10 \hat{i}) \text{ in/s}]$$

$$\mathbf{a}_A = (2.4 \hat{i} - 50 \hat{j}) \text{ in/s}^2$$

Then, find the internal angles of the mechanism at the given instant

$$l = \sqrt{(6 \text{ in})^2 + (2 \text{ in})^2} = 2\sqrt{10} \text{ in}$$

Using the law of sines:

$$\frac{\sin \theta}{4 \text{ in}} = \frac{\sin(180^\circ - 2\theta)}{2\sqrt{10} \text{ in}}$$

$$\theta = 37.8^\circ$$

and

$$\phi = 180^\circ - 2\theta = 104.5^\circ$$

$$\beta = \tan^{-1}\left(\frac{2 \text{ in}}{6 \text{ in}}\right) = 18.43^\circ$$

$$\varphi = 180^\circ - \phi - (\theta - \beta) = 56.13^\circ$$

$$\delta = \theta - \beta = 19.37^\circ$$

Then, find the velocity of point B

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{B/A} = \mathbf{V}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{V}_B = [(10 \hat{i}) \text{ in/s}] + \{(\omega_{AB} \hat{k}) \times (4 \text{ in})[\cos(19.37^\circ) \hat{i} + \sin(19.37^\circ) \hat{j}]\}$$

$$\mathbf{V}_B = (10 - 1.33\omega_{AB})\hat{i} + (3.8\omega_{AB})\hat{j} \quad \text{Eq. (1)}$$

Moreover, the velocity of point B can be found from linkage BO_4

$$\mathbf{V}_B = \boldsymbol{\omega}_{B/O_4} \times \mathbf{r}_{B/O_4}$$

$$\mathbf{V}_B = (\omega_{B/O_4} \hat{k}) \times \{(4 \text{ in}) \times [-\cos(56.13^\circ) \hat{i} + \sin(56.13^\circ) \hat{j}]\}$$

$$\mathbf{V}_B = (-3.32\omega_{B/O_4})\hat{i} + (-2.23\omega_{B/O_4})\hat{j} \quad \text{Eq. (2)}$$

Setting Eq. (1) equal to Eq. (2) in their respective directions

$$(10 - 1.33\omega_{AB})\hat{i} = (-3.32\omega_{B/O_4})\hat{i}$$

$$-1.33\omega_{AB} + 3.32\omega_{B/O_4} = -10$$

and

$$(3.8\omega_{AB})\hat{j} = (-2.23\omega_{B/O_4})\hat{j}$$

$$3.8\omega_{AB} + 2.23\omega_{B/O_4} = 0$$

Solving for the system of equations yields

$$\omega_{B/O_4} = -2.44 \text{ rad/s}$$

$$\omega_{AB} = 1.44 \text{ rad/s}$$

To find the angular acceleration of linkage BO_4 , first find the acceleration of point B

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_{A,n} + \mathbf{a}_{A,t} + \mathbf{a}_{B/A,n} + \mathbf{a}_{B/A,t}$$

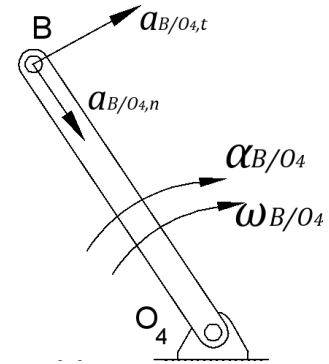
$$\mathbf{a}_B = \mathbf{a}_{A,n} + \mathbf{a}_{A,t} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}) + (\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A})$$

$$\mathbf{a}_B = [(2.4\hat{i} - 50\hat{j}) \text{ in/s}^2] + [(2.706\hat{k}) \text{ rad/s}] \times \{(4 \text{ in})[\cos(19.37^\circ) \hat{i} + \sin(19.37^\circ) \hat{j}]\} + (\alpha_{AB} \hat{k}) \times \{(4 \text{ in})[\cos(19.37^\circ) \hat{i} + \sin(19.37^\circ) \hat{j}]\}$$

$$\mathbf{a}_B = (-25.23 - 1.33\alpha_{AB})\hat{i} + (-59.714 + 3.77\alpha_{AB})\hat{j} \quad \text{Eq. (3)}$$

Similarly, find an expression for the acceleration of point B from linkage BO_4

$$\begin{aligned}
 \mathbf{a}_B &= \mathbf{a}_{B/O_4,n} + \mathbf{a}_{B/O_4,t} \\
 \mathbf{a}_B &= \boldsymbol{\omega}_{B/O_4} \times (\boldsymbol{\omega}_{B/O_4} \times \mathbf{r}_{B/O_4}) + (\boldsymbol{\alpha}_{B/O_4} \times \mathbf{r}_{B/O_4}) \\
 \mathbf{a}_B &= [(-4.6) \text{ rad/s}] \\
 &\quad \times [(-4.6 \hat{k}) \text{ rad/s}] \\
 &\quad \times \{(4 \text{ in})[-\cos(56.13^\circ) \hat{i} + \sin(56.13^\circ) \hat{j}]\} + (\alpha_{B/O_4} \hat{k}) \\
 &\quad \times \{(4 \text{ in})[-\cos(56.13^\circ) \hat{i} + \sin(56.13^\circ) \hat{j}]\} \\
 \mathbf{a}_B &= (47.2 - 3.32\alpha_{B/O_4}) \hat{i} + (-70.28 - 2.23\alpha_{B/O_4}) \hat{j} \quad \text{Eq. (4)}
 \end{aligned}$$



Setting Eq. (3) equal to Eq. (4) in their respective directions

$$\begin{aligned}
 (-25.23 - 1.33\alpha_{AB}) \hat{i} &= (47.2 - 3.32\alpha_{B/O_4}) \hat{i} \\
 -1.33\alpha_{AB} + 3.32\alpha_{B/O_4} &= 72.43
 \end{aligned}$$

and

$$\begin{aligned}
 (-59.714 + 3.77\alpha_{AB}) \hat{j} &= (-70.28 - 2.23\alpha_{B/O_4}) \hat{j} \\
 3.77\alpha_{AB} + 2.23\alpha_{B/O_4} &= -10.57
 \end{aligned}$$

Solving for the system of equations yields

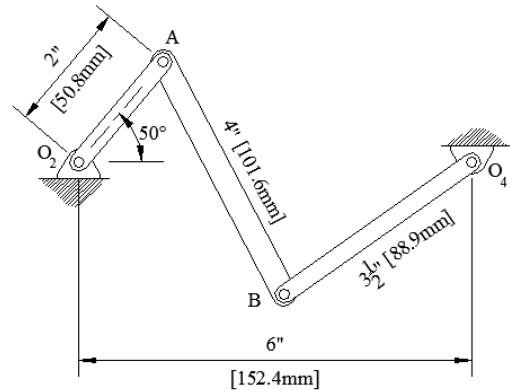
$$\begin{aligned}
 \alpha_{AB} &= -12.7 \text{ rad/s}^2 \\
 \alpha_{B/O_4} &= 16.73 \text{ rad/s}^2
 \end{aligned}$$

Answer: $\omega_{B/O_4} = 2.44 \text{ rad/s CW}$
 $\alpha_{B/O_4} = 16.73 \text{ rad/s}^2 \text{ CCW}$

Problem 2.16

Given Information:

The linkage mechanism shown is driven by link O_2A . At the instant shown the driving link is rotating at 8 radians per second clockwise and accelerating at 2.5 radians per second squared.



Required:

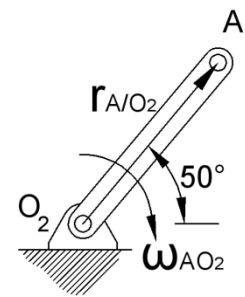
Determine the rotational velocity and acceleration of links AB and O_4B in the position shown.

Solution:

First, find the velocity of point A at the instant shown

$$\mathbf{V}_A = \omega_{AO_2} \times \mathbf{r}_{A/O_2} = [(-8 \hat{k}) \text{ rad/s}] \times \{(2 \text{ in})[\cos(50^\circ) \hat{i} + \sin(50^\circ) \hat{j}]\}$$

$$\mathbf{V}_A = (12.26 \hat{i} - 10.28 \hat{j}) \text{ in/s}$$



Then, find the internal angles of the linkage mechanism at the instant shown.

Applying the law of cosines to triangle O_2AO_4 :

$$l^2 = (2 \text{ in})^2 + (6 \text{ in})^2 - 2(2 \text{ in})(6 \text{ in}) \cos(50^\circ)$$

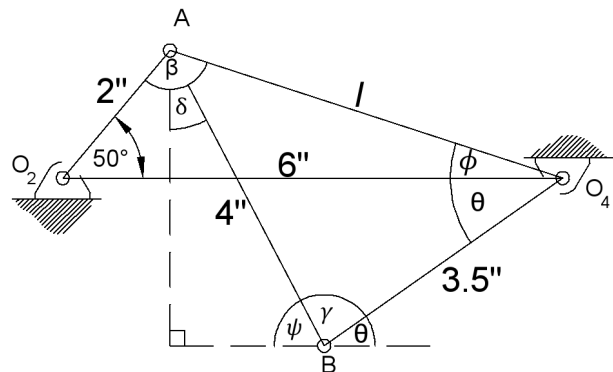
$$l = 4.96 \text{ in}$$

Applying the law of sines to triangle O_2AO_4 :

$$\frac{6 \text{ in}}{\sin(\beta)} = \frac{l}{\sin(50^\circ)} = \frac{2 \text{ in}}{\sin(\phi)}$$

$$\frac{4.96 \text{ in}}{\sin(50^\circ)} = \frac{2 \text{ in}}{\sin(\phi)}$$

$$\phi = 18^\circ$$



Applying the law of cosines to triangle BAO_4 :

$$(4 \text{ in})^2 = (3.5 \text{ in})^2 + (4.96 \text{ in})^2 - 2(3.5 \text{ in})(4.96 \text{ in}) \cos(\phi + \theta)$$

$$\phi + \theta = 53.1^\circ$$

$$\theta = 35.1^\circ$$

Applying the law of sines to triangle BAO_4 :

$$\frac{4 \text{ in}}{\sin(\theta + \phi)} = \frac{l}{\sin(\gamma)}$$

$$\frac{4 \text{ in}}{\sin(53.1^\circ)} = \frac{4.96 \text{ in}}{\sin(\gamma)}$$

$$\gamma = 82.6^\circ$$

and

$$\gamma + \psi + \theta = 180^\circ$$

$$\psi = 62.3^\circ$$

$$\delta = 90^\circ - \psi = 27.7^\circ$$

Then, determine the velocity of point B

$$\begin{aligned} \mathbf{V}_B &= \mathbf{V}_A + \mathbf{V}_{B/A} = \mathbf{V}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} \\ \mathbf{V}_B &= \mathbf{V}_A + \boldsymbol{\omega}_{AB} \times [(r_{B/A})(\cos \psi \hat{i} - \sin \psi \hat{j})] \\ \mathbf{V}_B &= [(12.26 \hat{i} - 10.28 \hat{j}) \text{ in/s}] + (\omega_{AB} \hat{k}) \times \{(4 \text{ in})[\cos(62.3^\circ) \hat{i} - \sin(62.3^\circ) \hat{j}]\} \\ \mathbf{V}_B &= (12.26 + 3.55\omega_{AB})\hat{i} + (-10.28 + 1.83\omega_{AB})\hat{j} \quad \text{Eq. (1)} \end{aligned}$$

The velocity of point B can also be calculated from linkage BO_4 as follows

$$\begin{aligned} \mathbf{V}_B &= \boldsymbol{\omega}_{BO_4} \times \mathbf{r}_{B/O_4} = (\omega_{BO_4} \hat{k}) \times [(r_{B/O_4})(-\cos \theta \hat{i} - \sin \theta \hat{j})] \\ \mathbf{V}_B &= (\omega_{BO_4} \hat{k}) \times \{(3.5 \text{ in})[-\cos(35.1^\circ) \hat{i} - \sin(35.1^\circ) \hat{j}]\} \\ \mathbf{V}_B &= (2.02\omega_{BO_4})\hat{i} + (-2.86\omega_{BO_4})\hat{j} \quad \text{Eq. (2)} \end{aligned}$$

Setting Eq. (1) equal to Eq. (2) in their respective directions

$$\begin{aligned} (12.26 + 3.55\omega_{AB})\hat{i} &= (2.02\omega_{BO_4})\hat{i} \\ 3.55\omega_{AB} - 2.02\omega_{BO_4} &= -12.26 \end{aligned}$$

and

$$\begin{aligned} (-10.28 + 1.83\omega_{AB})\hat{j} &= (-2.86\omega_{BO_4})\hat{j} \\ 1.83\omega_{AB} + 2.86\omega_{BO_4} &= 10.28 \end{aligned}$$

Solving for the system of equations yields

$$\begin{aligned} \omega_{AB} &= -1.033 \text{ rad/s} \\ \omega_{BO_4} &= 4.25 \text{ rad/s} \end{aligned}$$

Furthermore, substituting ω_{BO_4} into Eq. (2) to determine the velocity of point B yields

$$\begin{aligned} \mathbf{V}_B &= [2.02(4.25)]\hat{i} + [-2.86(4.25)]\hat{j} \\ \mathbf{V}_B &= [(8.59)\hat{i} + (-12.17)\hat{j}] \text{ in/s} \end{aligned}$$

Next, find the acceleration of point A

$$\begin{aligned} \mathbf{a}_A &= \mathbf{a}_{A,n} + \mathbf{a}_{A,t} = \boldsymbol{\omega}_{AO_2} \times \mathbf{V}_A + \alpha_{AO_2} \times \mathbf{r}_{A/O_2} \\ \mathbf{a}_A &= [(-8\hat{k}) \text{ rad/s}] \times [(12.26 \hat{i} - 10.28 \hat{j}) \text{ in/s}] + [(-2.5\hat{k}) \text{ rad/s}^2] \\ &\quad \times \{(2 \text{ in})[\cos(50^\circ) \hat{i} + \sin(50^\circ) \hat{j}]\} \\ \mathbf{a}_A &= (-78.41\hat{i} - 101.3\hat{j}) \text{ in/s}^2 \end{aligned}$$

Then, find an expression for the acceleration of point B