

CHAPTER 1

Problem 1:

(a) Trapezoidal channel with side slopes m_1 and m_2

$$A = (b + m_1 \frac{y}{2} + m_2 \frac{y}{2})y$$

$$T = b + y(m_1 + m_2)$$

$$P = b + y\sqrt{1 + m_1^2} + y\sqrt{1 + m_2^2}$$

$$D = A / T$$

$$R = A / P$$

(b) Trapezoidal channel with one vertical side

Set $m_1 = 0$ and $m_2 = m$ in the equations given in part (a).

(c) Right triangular channel

Set $m_1 = 0$, $b = 0$, and $m_2 = m$ in the equations given in part (a).

Problem 2:

(a) From the problem statement:

$$\gamma = 62.4 \text{ lbf/ft}^3$$

$$Y_c = \text{depth to centroid} = 2 \text{ ft}$$

$$A = d (1 \text{ ft/ft}) = 4 \text{ ft} (1 \text{ ft/ft}) = 4 \text{ ft}^2/\text{ft}$$

By using Equation 1.9

$$F_p = \gamma Y_c A = 62.4(2)(4) = 499 \text{ lbf / foot}$$

The hydrostatic pressure force is normal to the vertical sidewall.

(b) From the problem statement:

$$\gamma = 62.4 \text{ lbf/ft}^3$$

$$Y_c = \text{depth to centroid} = 2 \text{ ft}$$

$$A = (d^2 + (md)^2)^{1/2} (1 \text{ ft/ft}) = (4^2 + (2 \times 4)^2)^{1/2} \text{ ft} (1 \text{ ft/ft}) = 8.94 \text{ ft}^2/\text{ft}$$

By using Equation 1.9

$$F_p = \gamma Y_c A = 62.4(2)(4) = 1116 \text{ lbf / foot}$$

The hydrostatic pressure force is normal to the inclined side.

Problem 3:

$$v = 2.5v_* \ln\left(\frac{30z}{k_s}\right)$$

Let $k = k_s/30$. Then

$$v = 2.5v_* \ln\left(\frac{z}{k}\right)$$

Define q = discharge per unit width. By definition (see Equation 1.2)

$$q = \int_k^y v dz = \int_k^y \left[2.5v_* \ln\left(\frac{z}{k}\right) \right] dz = \int_k^y [2.5v_* \ln z] dz - \int_k^y [2.5v_* \ln k] dz$$

$$q = 2.5v_* [z \ln z - z]_k^y - 2.5v_* [z \ln k]_k^y = 2.5v_* \left[y \ln \frac{y}{k} - (y - k) \right]$$

Problem 4:

Noting that q = discharge per unit width and $k = k_s/30$ as in Problem 3, by definition (see Equation 1.3)

$$V = \frac{q}{y - k} = 2.5v_* \left[\frac{y \ln \frac{y}{k} - (y - k)}{y - k} \right] = 2.5v_* \left[\frac{y \ln \frac{y}{k}}{y - k} - 1 \right]$$

Because $y \gg k$, we have $(y - k) \approx y$. Therefore

$$V = 2.5v_* \left[\ln \frac{y}{k} - 1 \right]$$

Problem 5:

For the velocity distribution given, obviously, the velocity is maximum at the free surface. Substituting $z = y$ in

$$v = 2.5v_* \ln\left(\frac{30z}{k_s}\right)$$

we obtain

$$v_{\max} = 2.5v_* \ln\left(\frac{30y}{k_s}\right) = 2.5v_* \ln\left(\frac{y}{k}\right)$$

with $k = k_s/30$.

Problem 6:

Using the results of Problems 4 and 5

$$\frac{v_{\max}}{V} - 1 = \frac{2.5v_* \ln\left(\frac{y}{k}\right)}{2.5v_* \left[\ln \frac{y}{k} - 1 \right]} - 1 = \frac{1}{\ln \frac{y}{k} - 1}$$

with $k = k_s/30$.

Problem 7:

We found an expression for V in Problem 4. Here we will determine the value of z for which $v = V$. In other words with $k = k_s/30$

$$2.5v_* \ln\left(\frac{z}{k}\right) = 2.5v_* \left[\ln \frac{y}{k} - 1 \right]$$

$$\ln\left(\frac{z}{k}\right) = \left[\ln \frac{y}{k} - 1 \right] = \ln\left(\frac{y}{k}\right) - \ln(2.718) = \ln\left(\frac{y/k}{2.718}\right)$$

and

$$z = \frac{y}{2.718} = 0.37y$$

The point velocity at distance $0.37y$ from the bottom or $0.63y$ from the free surface will be equal to the cross sectional average velocity. Therefore, the velocity measured at $0.6y$ from the surface will be a good approximation to the average velocity.

Problem 8:

With $k = k_s/30$, we have

$$v = 2.5v_* \ln\left(\frac{z}{k}\right)$$

By definition

$$\beta = \frac{1}{V^2 A} \int v^2 dA$$

Using the velocity distribution given, for unit width, we can write

$$\beta = \frac{(2.5v_*)^2}{V^2 (y - k)} \int_k^y \ln^2\left(\frac{z}{k}\right) dz$$

Let us first evaluate the integration

$$\int_k^y \ln^2\left(\frac{z}{k}\right) dz = \int_k^y (\ln z - \ln k)(\ln z - \ln k) dz = \int_k^y (\ln^2 z - 2 \ln z \ln k + \ln^2 k) dz$$

$$\int_k^y \ln^2\left(\frac{z}{k}\right) dz = \left[z \ln^2 z \right]_k^y - 2 \int_k^y \ln z dz - 2 \ln k \int_k^y \ln z dz + \ln^2 k \int_k^y dz$$

$$\int_k^y \ln^2\left(\frac{z}{k}\right) dz = \left[z \ln^2 z - 2(z \ln z - z) - 2 \ln k (z \ln z - z) + \ln^2 k z \right]_k^y$$

$$\int_k^y \ln^2\left(\frac{z}{k}\right) dz = 2(y - k) + y \ln^2\left(\frac{y}{k}\right) - 2y \ln\left(\frac{y}{k}\right) \quad (*)$$

Also from Problem 4

$$V = 2.5v_* \left[\ln \frac{y}{k} - 1 \right]$$

$$V^2 = (2.5v_*)^2 \left[\ln \frac{y}{k} - 1 \right]^2 = (2.5v_*)^2 \left[\ln^2 \frac{y}{k} - 2 \ln \frac{y}{k} + 1 \right]$$

With $y \gg k$ and $(y-k) \approx y$

$$V^2(y - k) = (2.5v_*)^2 y \left[\ln^2 \frac{y}{k} - 2 \ln \frac{y}{k} + 1 \right] \quad (**)$$

Substituting Equations (*) and (**) into the expression for β

$$\beta = \frac{\left[(2.5v_*)^2 \right] 2(y - k) + y \ln^2\left(\frac{y}{k}\right) - 2y \ln\left(\frac{y}{k}\right)}{(2.5v_*)^2 y \left[\ln^2 \frac{y}{k} - 2 \ln \frac{y}{k} + 1 \right]}$$

Noting $(y-k) \approx y$ and simplifying

$$\beta = \frac{y + y \left[1 + \ln^2\left(\frac{y}{k}\right) - 2 \ln\left(\frac{y}{k}\right) \right]}{y \left[\ln^2 \frac{y}{k} - 2 \ln \frac{y}{k} + 1 \right]} = 1 + \frac{1}{\left[\ln \frac{y}{k} - 1 \right]^2}$$

From Problem 6

$$\frac{v_{\max}}{V} - 1 = \frac{1}{\ln \frac{y}{k} - 1}$$

Substituting this into the expression for β and rearranging

$$\beta = 1 + \left[\frac{v_{\max}}{V} - 1 \right]^2$$

Problem 9:

$$v_* = \left(\frac{\tau_0}{\rho} \right)^{1/2} = \left(\frac{3.7}{1000} \right)^{1/2} = 0.061 \text{ m/s}$$

From Problem 4

$$V = 2.5v_* \left[\ln \frac{y}{k} - 1 \right] = 2.5v_* \left[\ln \frac{30y}{k_s} - 1 \right] = (2.5)(0.061) \left[\ln \frac{30(0.94)}{(0.001)} - 1 \right] = 1.41 \text{ m/s}$$

$$q = \left[y - \frac{k_s}{30} \right] V \approx yV = (0.94)(1.41) = 1.33 \text{ m}^2/\text{s}$$

From Problem 5

$$\frac{v_{\max}}{V} - 1 = \frac{1}{\ln \frac{30y}{k_s} - 1} = \frac{1}{\ln \frac{30(0.94)}{(0.001)} - 1} = 0.11$$

$$\beta = 1 + \left[\frac{v_{\max}}{V} - 1 \right]^2 = 1 + (0.11)^2 = 1.01$$

$$\text{Rate of momentum transfer} = \beta \rho q V = (1.01)(1000 \frac{\text{kg}}{\text{m}^3})(1.33 \frac{\text{m}^2}{\text{s}})(1.41 \frac{\text{m}}{\text{s}}) = 1894 \frac{\text{kgm}}{\text{s}^2} / \text{m}$$

$$\alpha = 1 + 3 \left[\frac{v_{\max}}{V} - 1 \right]^2 - 2 \left[\frac{v_{\max}}{V} - 1 \right]^3 = 1 + 3(0.11)^2 - 2(0.11)^3 = 1.03$$

$$\text{Rate of kinetic transfer} = \alpha \frac{\rho}{2} q V^2 = (1.03) \frac{1}{2} (1000 \frac{\text{kg}}{\text{m}^3})(1.33 \frac{\text{m}^2}{\text{s}})(1.41 \frac{\text{m}}{\text{s}})^2 = 1362 \frac{\text{Nm}}{\text{s}} / \text{m}$$

Problem 10:

$$A_1 = 1 \text{ m} \times 20 \text{ m} = 20 \text{ m}^2$$

$$A_2 = 4 \text{ m} \times 4 \text{ m} = 16 \text{ m}^2$$

$$A_3 = 0.5(1 \text{ m} \times 22 \text{ m}) = 11 \text{ m}^2$$

$$Q = V_1 A_1 + V_2 A_2 + V_3 A_3 = 0.5(20) + 1.5(16) + 0.3(11) = 37.3 \text{ m}^3 / \text{s}$$

$$V = \frac{Q}{A} = \frac{Q}{A_1 + A_2 + A_3} = \frac{37.3}{20 + 16 + 11} = 0.79 \text{ m/s}$$

Problem 11:

From Equation 1.15

$$\beta = \frac{V_1^2 A_1 + V_2^2 A_2 + V_3^2 A_3}{V^2 A} = \frac{0.5^2(20) + 1.5^2(16) + 0.3^2(11)}{0.79^2(47)} = 1.43$$

Then by using Equation 1.11 with $\rho = 1000 \text{ kg/m}^3$

$$\text{Rate of momentum transfer} = \beta \rho Q V = 1.43(1000)(37.3)(0.79) = 42,138 \text{ kg} \cdot \text{m} / \text{s}^2$$

Likewise, from Equation 1.21

$$\alpha = \frac{V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3}{V^3 A} = \frac{0.5^3(20) + 1.5^3(16) + 0.3^3(11)}{0.79^3(47)} = 2.45$$

Then by using Equation 1.19

$$\text{Rate of kinetic energy transfer} = \alpha \frac{\rho}{2} Q V^2 = 2.45 \left(\frac{1000}{2} \right) (37.3)(0.79)^2 = 28,517 \text{ kg} \cdot \text{m} / \text{s}^3$$

Problem 12:

$$V = \frac{Q}{A} = \frac{Q}{y(b + my)} = \frac{100}{3.15(5 + 2 \times 3.15)} = \frac{100}{35.6} = 2.81 \text{ fps}$$

$$R = \frac{A}{P} = \frac{y(b + my)}{b + 2y\sqrt{1 + m^2}} = \frac{3.15(5 + 2 \times 3.15)}{5 + 2(3.15)\sqrt{1 + 2^2}} = \frac{35.6}{19.1} = 1.86 \text{ ft}$$

$$D = \frac{y(b + my)}{b + 2my} = \frac{3.15(5 + 2 \times 3.15)}{5 + 2 \times 2 \times 3.15} = \frac{35.6}{17.6} = 2.02 \text{ ft}$$

(a) Using Equation 1.23

$$R_e = \frac{4VR}{\nu} = \frac{4VR}{\nu} = \frac{4(2.81)(1.86)}{1.217 \times 10^{-5}} = 1,925,000$$

The flow is turbulent.

(b) Using Equation 1.24

$$F_r = \frac{V}{\sqrt{gD}} = \frac{2.81}{\sqrt{32.2 \times 2.02}} = 0.35$$

The flow is subcritical.

Problem 13:

- (a) Nonuniform
- (b) Nonuniform
- (c) Uniform
- (d) Nonuniform

Problem 14:

- (a) Unsteady
- (b) Steady
- (c) Steady

Problem 15:

Verify that

$$\frac{\partial(A Y_C)}{\partial x} = A \frac{\partial y}{\partial x}$$

Consider a horizontal strip of flow area having a length of b and thickness of dz and extending from one side of a flow section to the other side. Suppose the vertical distance between the centroid of this strip and the channel bottom is z . Then by definition

$$\frac{\partial(A Y_C)}{\partial x} = \frac{\partial}{\partial x} \int_0^y (y - z) b dz$$

Using the Leibnitz rule, we can write this as

$$\frac{\partial(A Y_C)}{\partial x} = \int_0^y \frac{\partial}{\partial x} (y - z) b dz + (y - y) b dz \frac{\partial y}{\partial x} - (0 - z) b dz \frac{\partial 0}{\partial x}$$

The last two terms on the right hand side are zero. Then

$$\frac{\partial(A Y_C)}{\partial x} = \int_0^y \frac{\partial}{\partial x} (y - z) b dz = \int_0^y \frac{\partial}{\partial x} (y b dz) - \int_0^y \frac{\partial}{\partial x} (z b dz)$$

Since z and b are not functions of x at a given section, the last term on the right hand side is zero. Also, since y is not a function of z and b and dz are not functions of x

$$\frac{\partial(A Y_C)}{\partial x} = \int_0^y \frac{\partial}{\partial x} (y b dz) = \frac{\partial y}{\partial x} \int_0^y b dz = A \frac{\partial y}{\partial x}$$